Uncertainty, Risk Aversion
and International Trade

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Abstract

This paper develops a general equilibrium model of international trade in homogenous intermediate inputs. In the model, trade between countries is driven by uncertainty in the delivery of inputs. Because their managers are risk-averse, final good firms contract with multiple suppliers to decrease the variability of their profits. The analysis shows that risk diversification provides an incentive for international trade over and above such reasons as comparative advantage and economies of scale, and highlights a new channel – a reduction in uncertainty – through which trade can increase welfare. Econometric evidence reveals that the model is consistent with qualitative features of the data.

Keywords: Intermediate inputs, international trade, trade liberalization, uncertainty.

JEL Classification Numbers: F1.
1 Introduction

Risk plays a relatively small role in international trade theory. In both the classic models of comparative advantage and the new trade theories, which feature economies of scale and product differentiation, cost reduction underlies the incentive to trade.\(^1\) Admittedly, there are studies that extend these models to include uncertainty; a literature dating back to the 1970s explores the conditions under which the predictions of the comparative advantage models of trade carry over to stochastic environments (e.g., Turnovsky (1974); Helpman and Razin (1978); or Helpman (1988)) while more recent papers extend new trade models to study the effects of uncertainty on the export decisions of firms, the production location decisions of multinational enterprises, and the effect of trade on income volatility (e.g., de Sousa et al. (2015); Ramondo et al. (2013); Caselli et al. (2015); Fillat and Garetto (2015); Fillat et al. (2015); and Esposito (2016)). On the whole, however, it seems reasonable to say that risk as an impetus to international trade, absent of other motives for trade, has received little attention from formal trade theory.

The marginal role of risk is surprising in light of the following facts. First, efficient management of supply-chain risk has long been recognized as an important determinant of firm performance. Because delays in materials flows lead to increased costs, sales losses, and ultimately lower profits, firms often source the same input from multiple suppliers and are willing to trade-off input cost against its variability when making sourcing decisions.\(^2\) Second, the rise in the value of world trade is mainly due to the vertical disintegration of production and the extensive cross-shipping of components associated with global supply chains (e.g., Feenstra (1998); Hummels et al. (2001); and Timmer et al. (2014)). According to recent estimates, intermediate inputs now account for as much as two-thirds of international trade flows (e.g., Johnson and Noguera (2012)). Together, these considerations suggest that supply-chain risk management potentially plays an important role in explaining trade patterns.

\(^1\)In classic theories, trade reduces costs by allowing countries to specialize according to comparative advantage arising from differences in productivity (e.g., Ricardo (1819); Dornbusch et al. (1977); or Eaton and Kortum (2002)) or resource endowment (e.g., Heckscher (1919) and Ohlin (1933)). In new trade theories, trade enables firms to benefit from economies of scale, thereby decreasing average production costs and increasing welfare (e.g., Krugman (1980) and Melitz (2003)). I note that Brander and Krugman (1983) develop a model in which the rivalry of oligopolistic firms serves as an independent cause of international trade.

\(^2\)Antràs et al. (2014) find that U.S. manufacturing firms import narrowly defined inputs from about 3 sources on average (the 95\(^{th}\) percentile is 11). Gervais (2016) reports that the U.S. purchase narrowly defined homogenous intermediate products from about 7.5 sources on average and finds a statistically significant negative association between input price variability and import demand, after controlling for expected input price. Chopra and Sodhi (2004), Szwejczewski et al. (2005), and Tang and Musa (2011) also provide evidence of multi-sourcing.
Anecdotal evidence supports this conjecture. For example, in 2012, research firm UBM TechInsights took apart several of Apple’s iPads and found components with the same functions made by at least three manufacturers in different tablets (Clark (2012)). The teardown revealed not only the breadth of suppliers, but also that the suppliers’ main establishments are located in different geographic regions. As another example, the 2011 tsunami in Japan and flooding in Thailand caused severe supply-chain disruptions in a number of industries, especially the automotive and electronics industries (e.g., Fuller (2011) and Dawson (2011)). In response, many major manufacturers are now more actively practicing supply risk mitigation. For instance, prior to the tsunami, automakers were sourcing the vast majority of their micro-controllers from Japanese semiconductor giant Renesas. Following the tsunami, they began to look for additional suppliers outside Japan; Freescale, a U.S. company, stepped in and currently supplies about 22 percent of automotive chips (Greimel (2014)).

Empirical studies also provide evidence of a link between uncertainty and sourcing decisions. Wolak and Kolstad (1991) estimate a model of input demand using data on Japanese imports of steam-coal from five countries for the period 1983 to 1987. According to their estimates, Japan is willing to pay 29 to 50 percent above the current market price for a supply of coal having no price risk. This result helps rationalize the fact that the share of Japanese steam-coal imports from Australia is consistently more than double that from South Africa despite the mean price over the sample period being about the same for both countries. While Wolak and Kolstad (1991) restrict their study to steam-coal, sourcing inputs from multiple countries seems quite common. For example, Figure 1 presents the distribution of homogenous intermediate product categories over the number of countries from which U.S. firms imports each product. The figure makes clear that in most cases the U.S. imports the same input from more than one country (the median number of countries is about 7 and the mean 9). Because these are homogenous product categories, it is difficult to appeal to comparative advantage or product differentiation to explain these sourcing patterns.

The current paper starts from the premise that the benefits of multi-sourcing – the strategy of buying the same input from multiple suppliers – are similar to those of portfolio diversification in theoretical finance: an increase in the number of geographically diverse suppliers reduces the variability of profits, much like an increase in the number of assets with imperfectly correlated returns reduces the variance of a portfolio’s return (e.g., Markowitz (1952) and Sharpe (1964)). Therefore, I adapt methods derived from modern portfolio

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3 In addition to natural disasters, labor dispute, supplier bankruptcy, acts of wars and terrorism are also important sources of uncertainty (e.g., Sheffi (2001); or Jüttner (2005)).

4 Other explanations for multi-sourcing have been provided such as capacity constraints, entry deterrence, bargaining power, and hold-up problems (e.g., Tomlin (2006); Burke et al. (2007); or Mukherjee and Tsai
theories to develop a general equilibrium model of firm decisions which rationalizes multi-sourcing strategies. I then use the theoretical model to study the role of supply-chain risk management in explaining sourcing decisions and, more broadly, international trade patterns. Among other results, the analysis shows that risk diversification provides an incentive for trade over and above such reasons as comparative advantage and economies of scale, and highlights a new channel – a reduction in uncertainty – through which trade can increase welfare.

For the analysis, I model risk as unexpected variation in output delivery originating from supplier-level productivity shocks. This captures in a simple way the fundamental impact of a broad range of potential events associated with supply-chain risk (e.g., increase in production costs, delayed shipments, or low quality inputs). To study the effects of risk on sourcing decisions, I assume that final good firms must contract materials before the uncertainty is resolved and that there is no derivatives market for inputs. Managers’ risk-aversion provides an incentive for firms to diversify away input uncertainty by contracting with multiple suppliers. In equilibrium, firms select a portfolio of suppliers and a distribution of input demand across these suppliers that optimally trades off expected profits with variability.

In the model, suppliers’ productivity shocks can be decomposed into idiosyncratic and country-specific components. Together, these components govern the dispersion of realized production costs across suppliers within each country as well as the correlation between suppliers’ production costs. In a closed economy, a multi-sourcing strategy will only reduce the impact of the idiosyncratic components of productivity shocks. In contrast, firms in an open economy can simultaneously diversify away the idiosyncratic and the country-specific components of risk by purchasing inputs from domestic and foreign suppliers. The analysis shows that, because trade provides access to more efficient diversification opportunities, a smaller share of resources is devoted to risk diversification activities (i.e., supplier-level fixed

(2013)). Compared to risk diversification, these studies represent a relatively small share of the literature on multi-sourcing.

5 To focus the analysis on the role of multisourcing, the model assumes away the other form of insurance available to firms. For example, there are deep stocks and currencies markets which allow firms to insure against unforeseen country-level productivity shocks and exchange rate fluctuations. There also exists well developed markets for selected commodities (e.g., energy, metals, grains, and livestock) and a few manufactured products (e.g., random-access memory) that can be used to hedge against industry-level shocks. Nevertheless, the incentive for multi-sourcing remains because these instruments do not provide insurance against country-industry risk (e.g., car parts from Japan) or supplier-level risk which form the core of the model. However, in view of these comments, the model should be interpreted as studying the impact of risk that is undiversifiable through an organized exchange.

6 An extensive literature shows that moral hazard and adverse selection issues create divergence between managers’ and shareholders’ interests and provide an incentive to shareholders to tie the value of managers’ compensation to the value of their firms. This type of compensation scheme prevents managers from diversifying firm-level risk to the extent that shareholders can (e.g., Lambert et al. (1991); or Murphy (1999)). Empirical studies provide evidence that companies are controlled by imperfectly diversified owners and, as a result, are risk-averse (e.g., Heaton and Lucas (2000); Faccio et al. (2011); or Lyandres et al. (2013) and references therein).
costs in the model) in an open economy. As a result, (expected) equilibrium output per worker, a natural measure of welfare, increases following trade liberalization.

The theoretical model provides several predictions that can be confronted with data. In this paper, I use information on disaggregated U.S. imports to test the empirical validity of two of the main implications of the theory related to firms’ sourcing decisions. First, I study the relationship between input demand and supplier characteristics. Consistent with the model, the empirical results show that U.S. firms purchase a larger fraction of their inputs from suppliers characterized with low expected price and low price variability. Second, I examine the relationship between industry characteristics and the distribution of input demand across suppliers. As predicted by the model, I find that industries characterized by more risk, measured as the variance of price “shocks”, have more dispersed demand. In other words, firms buy from more suppliers and spread their input demand more evenly across suppliers when there is more uncertainty.

The rest of the paper proceeds as follows. In the next section, I briefly review related studies. In section 3, I set out an analytical model of firm-level sourcing decisions under uncertainty. In section 4, I use the model to evaluate the effects of changes in uncertainty on equilibrium industry characteristics and welfare. In section 5, I extend the model to an arbitrary number of identical economies and study the impact of moving from autarky to free trade on equilibrium outcomes. In section 6, I evaluate the impact of a reduction in trade costs on the optimal sourcing strategy. In section 7, I present econometric evidence supporting the view that risk is an important determinant of bilateral trade patterns. Finally, in section 8, I present some concluding comments.

2 Literature

This is not the first paper to use a portfolio approach to model sourcing decisions. A few studies use modern portfolio theories as a basis to develop an empirical methodology to estimate the role of risk aversion in explaining sourcing decision, such as the study of Japanese steam-coal imports by Wolak and Kolstad (1991) discussed in the introduction. Other works include Appelbaum and Kohli (1997) who estimate oil and non-oil import demand functions for the United States to assess the impact of uncertainty on the volume of imports, and the distribution of income; Appelbaum and Kohli (1998), who estimate the effects of import-price uncertainty on factor income in Switzerland; and Muhammad (2012) who estimates carnation demand in the United Kingdom.7

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7Other studies use portfolio models in an international context. Allen and Atkin (2015) use a mean-variance model to show that, while trade liberalization increases the volatility of Indian farmers’ returns, the welfare cost of this increased volatility are dwarfed by the first moment gains from trade. di Giovanni and
While supply-chain-risk mitigation is not yet a prominent topic in international trade, there are important related theoretical literatures in the fields of logistic management and operational research. Typically, these papers use numerical methods to solve partial equilibrium models of a single firm choosing the optimal allocation of demand across a known set of suppliers (e.g., variants of the so-called “newsvendor” model). My approach contrasts with this literature in two important aspects. First, my model provides analytical expressions for both the optimal distribution of input demand across suppliers and the optimal set of suppliers (i.e., the number of suppliers and their distribution across countries). Second, I embed my sourcing decision framework into a general equilibrium model of international trade. These extensions allow me to study the impact of changes in uncertainty and trade barriers on optimal sourcing decisions, trade flows, and welfare.

This paper is related to several other strands of the literature. A number of recent studies have looked into the role of firm-level inventory adjustments and country-specific shocks in explaining trade flows between countries (e.g., Alessandria et al. (2010) and Novy and Taylor (2014)). These papers argue that the sharp decline in trade that followed the 2007-08 financial crisis was driven by a decrease in inventory holding in response to increased uncertainty. In contrast with these studies, I develop a model in which firms do not hold inventories and rely on deliveries from their suppliers to produce output. My approach is consistent with widespread lean manufacturing and just-in-time practices, and the fact that the U.S. auto industry all but halted when major Japanese suppliers were taken offline by the 2011 tsunami.

The paper is also related to the international sourcing literature (e.g., Antràs (2003); Antràs and Helpman (2004); and Grossman and Helpman (2004)). These studies show that contractual imperfections and distorted incentives of input providers, due to hold-up or agency problems, lead to production inefficiency. In response, final good firms choose a specific organizational form in an attempt to reduce this inefficiency. These papers assume each firm contracts with at most one supplier and concentrate on the “make-or-buy” decision. Instead, I assume away contractual imperfections and require that firms purchase inputs from suppliers. These assumptions focus the analysis on the impact of risk on the optimal sourcing strategy (i.e., the optimal set of suppliers and the distribution of input demand across suppliers).

Levchenko (2012) use a portfolio model to evaluate the risk content of countries’ exports. In the same spirit, Vannoorenberghe et al. (2014) use detailed firm-level Chinese data to evaluate the volatility of firms’ exports. Tang (2006) and Snyder et al. (2012) provide informative reviews of these literatures. Of note is the recent work of Martinez-de Albéniz and Simchi-Levi (2006), which uses a mean-variance model to study the trade-offs faced by a manufacturer signing a portfolio of option contracts with its suppliers and having access to a spot market.

See also Swenson (2005); Baldwin and Okubo (2014); and Gorodnichenko et al. (2015) for related studies.
The implications of foreign intermediate inputs on firm performance and aggregate productivity are attracting increasing attention. This includes both empirical studies such as Amiti and Konings (2007) for Indonesia, Goldberg et al. (2010) for India and Blaum et al. (2015) for France, and theoretical studies such as Rodríguez-Clare (2010), Garetto (2013), and Antrás et al. (2014). These studies contrast with previous works by focusing on intermediate input imports, as opposed to final goods exports. However, because they build on standard models, variation in input prices or input differentiation across countries remains the motive for trade. Instead, my model emphasizes supplier-level uncertainty as an impetus to trade. The theoretical analysis presented in this paper shows that a decrease in trade costs increases the demand for foreign inputs and decreases (expected) average production costs. These results are consistent with the main empirical findings of the intermediate inputs literature, which show that improved access to foreign inputs increases productivity.

3 Closed Economy Model

In this section, I develop a model of sourcing decisions under input delivery uncertainty that rationalizes the purchase of identical intermediate inputs from multiple suppliers.

3.1 Set-up of the model

Consider a closed economy composed of two types of producers: final good firms and suppliers of intermediate inputs. The production of the final good is subject to two technology constraints. First, it involves increasing returns to scale captured by a fixed set-up cost, equal to $F$ units of labor, that must be paid before production can start. The presence of a firm-level fixed cost provides an incentive to expand production and contract (potentially) with multiple suppliers. Second, once the fixed cost is paid, materials can be transformed at no further cost into final goods. For simplicity, I follow Antrás (2003) and choose physical units such that

$$q = M,$$

(1)

where $q$ denotes final good output and $M$ is the amount of materials.

To study sourcing decisions, I assume final good firms must purchase intermediate inputs from suppliers. Materials are produced using only one factor, labor. Production of intermediates also entails both fixed and marginal production costs. The fixed cost, denoted $f$, is deterministic and common to all suppliers. It reflects the resources devoted to preparing the workplace to produce materials that meet the specifications of the downstream firm. Because the fixed cost is specific to each downstream firm, there are no economies of scope and each supplier produces materials for a single final good firm. After the fixed cost is paid,
labor can be transformed at a constant rate $z$ into materials such that

$$m = zl,$$

where $m$ is the output of materials and $l$ is the labor used for production. The parameter $z$ is stochastic and varies across suppliers. Suppliers learn their productivity only after they begin production and the fixed cost, $f$, is sunk. Suppliers that receive favorable shocks (i.e., high $z$) are able to produce more materials with a given number of workers compared to suppliers with low productivity.

Uncertainty in productivity reflects aggregate shocks (e.g., natural disaster, or acts of war and terrorism) as well as idiosyncratic shocks (e.g., problems with machines or production defects). Let $\mathcal{S}$ denotes the set of (potential) suppliers in the economy. For simplicity, I assume that the expected productivity, the variance of productivity, and the correlation between productivities are common across suppliers and respectively given by

$$\mathbb{E}(z_k) = \mu, \text{ var}(z_k) = \sigma^2, \text{ and } \text{corr}(z_k, z_h) = \rho \in (0, 1), \forall k, h \in \mathcal{S}, k \neq h,$$

where $k, h \in \mathcal{S}$ index suppliers. The properties of the productivity distribution are common knowledge to all agents in the model.

Intermediate inputs must be contracted before the realization of uncertainty.\(^\text{10}\) Because understanding the impact of the contracting environment on the optimal sourcing decision is beyond the scope of this paper, I make a number of simplifying assumptions to focus the analysis on the impact of uncertainty. First, the quantity of materials delivered and the realized costs of production are observable by third parties. Second, there is no (ex post) spot market for inputs or, equivalently, the barriers to selling inputs on the spot market are prohibitive, such that inputs have no value outside the relationship.\(^\text{11}\) Third, the contract terms specify the distribution of conglomerate profits between final good firms and suppliers (i.e., profit sharing). For simplicity, I assume that suppliers break even in every state of the world, such that final good firms’ managers bear all the risk.\(^\text{12}\)

Before characterizing the optimal behavior of the representative final good firm, the key agent of the model, I briefly describe the demand side of the economy. By assumption,\(^\text{10}\) As in Antrás (2003) and Antrás and Chor (2013), there is an unbounded mass of ex ante identical (potential) suppliers. A random subset of these suppliers are matched to final good firms. Once they are matched, suppliers transact with only one final good firm.\(^\text{11}\) Organized exchanges of this type are the exception rather than the norm and generally account for a small fraction of overall sales (e.g., Seifert et al. (2004)).\(^\text{12}\) This is equivalent to assuming that final good firms own their suppliers and that managers maximize the welfare of the entire conglomerate. Another interpretation is that, as in Antrás and Chor (2013), suppliers can either engage in intermediate inputs production or in an alternative activity that provides zero profits. In that case, risk averse suppliers are indifferent between the outside option and producing materials for final good firms if the contract terms adjust so that they break even in every state of the world.
consumers have no taste for leisure and final goods are homogeneous. Therefore, because preferences are unique up to a monotonic transformation, any increasing function of consumption will be a candidate to characterize the preferences for the representative consumer. Further, because consumers always spend their entire income on final goods, aggregate demand is given by \( D = E/p \), where \( E \) denotes aggregate income and \( p \) is the price of final goods.

### 3.2 Final good firms’ behavior

Managers of final good firms decide how much to produce, the set of suppliers to contract with, and the amount of materials to order from each (or equivalently the distribution of employment across suppliers). Because final good firms’ managers bear all the risk, they take their production costs as well as the production costs of all of their suppliers into account when making decisions. Under the maintained assumptions, the total costs associated with a final good firm is given by

\[
\Gamma = w \left( F + nf + \sum_{k=1}^{n} l_k \right),
\]

where \( w \) denotes the wage rate (henceforth normalize to 1 without loss of generality), \( n \) is the number of suppliers from which the firm buys inputs, and \( l_k \) is the number of production workers employed by supplier \( k = 1, 2, ..., n \).

From equations (1) and (4), it follows that firm profits depend on the price of final goods, \( p \), and the distribution of labor as follows

\[
\pi = p \left( \sum_{k=1}^{n} z_k l_k \right) - \left( F + nf + \sum_{k=1}^{n} l_k \right).
\]

Equation (5) makes clear that profits are stochastic. While the distribution of employment is known \textit{ex ante} (because suppliers are contractually obligated to commit to a certain amount

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13 The analysis shows that risk aversion allows increasing returns to scale to be reconciled with perfect competition. This is, in a way, reminiscent of the contestable markets literature where increasing returns to scale and perfect competition are made consistent by the presence of an outside threat (e.g., Baumol et al. (1982)).

14 The total production costs function defined in equation (4) is an extension of functions commonly used in the international trade literature. First, suppose that there is no uncertainty. In that case, supplier-level fixed costs imply each final good firm will find it optimal to buy materials from a unique supplier. Replacing materials purchase with final good output into the total costs function (4) yields \( \Gamma = w (F + c) \), where \( F = F + f \) and \( c = 1/\mu \). This total costs function is the same as in new trade models (e.g., Krugman (1979) and Krugman (1980)). Second, adding uncertainty in marginal costs but limiting final good firms to only one supplier yields a total costs function that varies across firms as follows \( \Gamma_s = w (F + q/z_s) \). This cost structure is similar to that of heterogeneous firms model of trade (e.g., Melitz (2003)). A key distinction, however, is that in my model firms must make production decisions before the realization of the uncertainty.
of labor to the production of materials) the amount of material delivered is learned \textit{ex post} (once production has begun and productivity is revealed).\footnote{In Appendix A.1 at the end of the paper, I outline a model where contracts specify the quantity of materials to be delivered by suppliers (independent of their productivity draw). In that case, the uncertainty arises from the stochastic input costs. Suppliers hit by bad shocks will be forced to hire more workers to produce the contracted quantity of materials, which increases the average cost of delivered materials. Under reasonable assumptions, the cost uncertainty model yields the same optimal number of suppliers and distribution of input demand across suppliers as the input uncertainty model developed in the main text.}

Because managers are risk-averse and profits are unknown when decisions are made, they maximize the \textit{expected} utility of profits. The preferences of the final good firms’ managers are represented by a concave utility function \( U(\pi) \), with \( U'(\pi) > 0 \) and \( U''(\pi) < 0 \). Assuming that \( U(\pi) \) is continuously differentiable up to the second-order, a Taylor expansion of \( U(\pi) \) evaluated at \( E(\pi) \) is given by

\[
U(\pi) \approx U(\mathbb{E}(\pi)) + U'(\mathbb{E}(\pi)) \cdot [\pi - \mathbb{E}(\pi)] + \frac{1}{2} U''(\mathbb{E}(\pi)) \cdot [\pi - \mathbb{E}(\pi)]^2.
\]  

(6)

Taking expectations yields

\[
\mathbb{E}[U(\pi)] \approx U(\mathbb{E}(\pi)) + \frac{1}{2} U''(\mathbb{E}(\pi)) \cdot \text{var}(\pi).
\]  

(7)

This equation shows that expected utility of profits depends not only on the expected level of profits but also on the variance of profits. An important caveat of equation (7) is that it requires the full specification of the utility function.

From the expected utility theory, we know that maximizing the certainty equivalent provide the same solution as maximizing the expected utility (e.g., Eeckhoudt et al. (2005) and de Sousa et al. (2015)). To obtain the certainty equivalent, I first define the risk premium, \( \mathcal{P} \), as the amount of money that makes an agent indifferent between the risky return and the expected return:

\[
\mathbb{E}[U(\pi)] = U(\mathbb{E}(\pi)) + \mathcal{P} \cdot U'(\mathbb{E}(\pi)).
\]  

(8)

The first equality implicitly defines the risk premium and the last term is a first order Taylor approximation. Then, by combining equations (7) and (8), I can solve for the risk premium

\[
\mathcal{P} \approx - (\beta/2) \text{var}(\pi),
\]  

(9)

where \( \beta \equiv U''(\mathbb{E}(\pi)) / U'(\mathbb{E}(\pi)) \) denotes the manager’s marginal rate of substitution between expected profits and risk. Substituting with this results in equation (8), it follows that the
managers’ objective function can be approximated by

$$E[U(\pi)] \approx E(\pi) - (\beta/2) \text{var}(\pi),$$

(10)

This objective function is the same as that made for investors in classic portfolio selection models (e.g., Sharpe (1964), Lintner (1965) and Mossin (1966)). In the special cases where the utility function is quadratic or the productivity shocks have a multivariate normal distribution, the expression in equation (10) is exact (e.g., Samuelson (1970), Sargent (1979)) and Ingersoll (1987)). In general cases, the objective function is valid in the neighborhood of $E(\pi)$ and when the skewness, kurtosis, and other higher moments of the shock distributions are relatively small.\textsuperscript{16}

The final goods industry is perfectly competitive, such that managers maximize the expected utility of profits by choosing how much output to produce conditional on the market price.\textsuperscript{17} This involves two interrelated decisions. First, the firm chooses the set of suppliers with which to contract. Second, the firm chooses the allocation of input demand or, equivalently, employment across the selected suppliers. Because suppliers are identical \textit{ex ante} (i.e., at the moment contracts are signed), the optimal share of employment is constant across suppliers and given by $1/n$, where $n$ denotes the (endogenous) number of suppliers from which the firm buys inputs. Using this result and equation (5), it is straightforward to show that the variance of profits depends on the number ($n$) and size ($l$) of suppliers as follows

$$\text{var}(\pi) = \text{var} \left( p \sum_{k=1}^{n} z_k l_k \right) = (1 - \rho + n\rho) np^2 l^2 \sigma^2 = \left( 1 - \frac{\rho}{n} + \rho \right) n^2 \sigma^2,$$

(11)

\textsuperscript{16}As shown in equation (10), using a second-order Taylor expansion implies that managers have constant absolute risk aversion (CARA) preferences. This assumption gives a lot of tractability to the model and allows me to derive analytical solutions in a general equilibrium context. While it is straightforward to add higher terms to the expansion, analytical solutions quickly become intractable. This explain why most papers that study the role of risk in international trade do not go beyond the second moment. A notable exception is the partial equilibrium model of de Sousa et al. (2015), which features a third order polynomial approximation. In that case, the objective function includes the skewness of profits such that managers have decreasing absolute risk aversion (DARA).

\textsuperscript{17}I experimented with models that combine constant elasticity of substitution (CES) preferences and monopolistic competition. In this set-up, firms have market power and choose the profit maximizing price conditional on the downward slopping (residual) demand curve they face. As shown in new trade theories (e.g., Krugman (1980)), product differentiation provides an impetus to trade and introduces a new source of gains from trade: variety in consumption. Because I am interested in showing that risk is an independent motive for trade and provides a new channel through which trade can increase welfare, it is therefore preferable to assume final goods are homogenous.
where $\tilde{r} \equiv pnl$ is a measure of firm size.\textsuperscript{18} Equation (11) shows that, conditional on size, an increase in the number of suppliers reduces the variance of profits. Intuitively, when the firms contract with a single supplier (i.e., $n = 1$), the variance of profits is proportional to the variance of productivity, $\sigma^2$. As the number of suppliers goes to infinity (i.e., $n \to \infty$), idiosyncratic shocks are diversified away and final good firms face only the (undiversifiable) country-specific risk (i.e., the covariance between shocks, $\rho \sigma^2$).

Using these results, the manager’s problem can be expressed in terms of choosing the number of suppliers and the number of workers employed at each supplier as follows

$$\max_{n,l} \mathbb{E}[U(\pi)] = pn\mu l - (F + nf + nl) - (\beta/2) (1 - \rho + n\rho) np^2 l^2 \sigma^2.$$  (12)

The first term represents expected revenue, the second term is total labor costs, and the third term captures the impact of risk on the expected utility of profits. The firms’ problem defined in equation (12) is reminiscent of the classic portfolio choice decision with $n$ identical assets. There is one important distinction, however. In the portfolio literature, investors typically do not face transaction costs.\textsuperscript{19} As a result, they purchase shares of every available asset and the optimization problem reduces to choosing the optimal weights of each asset in the portfolio. Instead, in the current model final good firms face supplier-level fixed production costs and will not find it optimal to buy inputs from every potential supplier.

The objective function defined in equation (12) highlights a key mechanism of the model associated with risk. The disutility of production (the last two terms on the right hand side) is a function of the total labor costs and the variability of profits. As shown in Figure 1, the (expected) average cost of a unit of material ($F/n\mu l + f/\mu l + 1/\mu$) is monotonically decreasing in firm size and converges to the expected cost of a unit of material, $1/\mu$. However, because increasing input demand per supplier (i.e., raising employment $l$) increases the firm’s exposure to idiosyncratic shocks, and increasing the number of suppliers ($n$) raises fixed costs, the average disutility of production is U-shaped, as can be seen in Figure 2. This is an important property of the model which implies that, despite economies of scale, perfectly competitive firms will want to operate at the finite efficient scale (i.e., where average costs are at their minimum).\textsuperscript{20}

\textsuperscript{18}The variance of final goods price does not show up in equation (11) because, as explained below, the market price in the free entry equilibrium is independent of the realization of uncertainty.

\textsuperscript{19}There are exceptions such as Brennan (1975).

\textsuperscript{20}In the new trade theory beginning with Krugman (1979) firms face only labor costs and produce differentiated products. Because consumers have taste for variety, firms face a downward sloping demand curve which limits their size. Here, firms produce homogenous goods and face a perfectly elastic demand curve. Firm size is limited by the risk-aversion of managers. This happens because, under the maintained assumptions (i.e., CARA preferences), there are diseconomies of scale due to the increasing disutility of risk as the firm becomes larger.
The two first-order conditions for the problem defined in (12) are

\[
\frac{\partial \mathbb{E} [U(\pi)]}{\partial n} = 0 \quad \Leftrightarrow \quad p\mu l - f - l = \left(\frac{\beta}{2}\right)p^2\sigma^2l^2(1 - \rho + 2\rho n), \quad (13)
\]

\[
\frac{\partial \mathbb{E} [U(\pi)]}{\partial l} = 0 \quad \Leftrightarrow \quad p\mu - 1 = \beta p^2\sigma^2l(1 - \rho + n\rho). \quad (14)
\]

Equation (13) states that, conditional on employment per supplier \(l\), the marginal revenue from contracting with an additional supplier must be just equal to the marginal increase in risk associated with adding an extra supplier (and increasing expected output by \(\mu l\)). Equation (14) states that the marginal revenue from increasing employment at each supplier must be equal to the corresponding marginal increase in risk.\(^{21}\) Together, conditions (13) and (14) imply that firms operate at the efficient scale depicted in Figure 2.

Combining first-order conditions (13) and (14) provides an analytical expression for the optimal number of production workers per supplier

\[
l = \left[ \frac{2f}{\beta p^2(1 - \rho)\sigma^2} \right]^{\frac{1}{2}}. \quad (15)
\]

This equation shows that an increase in supplier’s fixed production costs \(f\) provides an incentive to concentrate input demand among fewer suppliers. It also shows that the optimal demand per supplier is increasing in the correlation between shocks, \(\rho\). When suppliers’ productivity shocks are strongly correlated (i.e., risk is mostly due to country-specific factors), there is little benefit from contracting with more than one supplier. Conversely, equation (15) shows that an increase in the variance of production costs \(\sigma^2\) or in the risk-aversion parameter \(\beta\) increases the incentive for firms to diversify risk (by contracting with a large number of suppliers) thereby decreasing the optimal demand per supplier.

### 3.3 Free entry equilibrium

There is an unbounded pool of identical prospective entrants into the final good industry and entry into the industry is unrestricted. As a result, firms will continue to enter the industry until the expected utility of profits is equal to zero. From (12), this free-entry condition

\(^{21}\)In making these statements, I have ignored the integer constraint. The equilibrium solution for some combination of parameters might be a fraction, in which case the optimal number of suppliers would be equal to the nearest feasible integer point. For simplicity, I restrict the analysis to cases where the first order conditions (13) and (14) are reasonable approximations. Other cases would require using integer programming methods which do not yield tractable analytical solutions. I note that interpreting \(n\) as the mass of suppliers (which would eliminate the need for the integer constraint) is not consistent with the definition of the variance of profits in equation (11). If downstream firms contract with a mass of suppliers, they completely diversify the idiosyncratic component of risk and face only the aggregate risk.
requires that
\[ pn\mu l - (F + nf + nl) = (\beta/2) \left(1 - \rho + n\rho \right) np^2 l^2 \sigma^2. \] (16)

This condition states that, in equilibrium, expected profits should compensate exactly for the risk borne by managers.

Combining the first-order condition (13) and the free-entry condition (16), and substituting in the equilibrium number of production workers per supplier, \(l\) defined in (15), provides an analytical expression for the optimal number of suppliers per final good firm
\[ n^* = \left[ \left(1 - \frac{\rho}{\rho} \right) \frac{F}{f} \right]^\frac{1}{2}. \] (17)

Equation (17) shows that, in equilibrium, the number of suppliers is increasing in the ratio of firm-level fixed production costs for final goods to supplier-level fixed production costs \((F/f)\). When firm-level fixed costs are large, it is optimal for final good firms to produce a great quantity of output in order to exploit returns to scale in final good production. As a result, an increase in firm-level fixed costs increases the optimal number of suppliers per firm. Conversely, an increase in supplier-level fixed costs reduces the net benefit from diversifying input demand and decreases the optimal number of suppliers per firms. Equation (17) also shows that the optimal number of suppliers per firm is decreasing in the correlation between suppliers shocks, \(\rho\). When the correlation is large, risk is mostly country-specific such that there is little motive for diversification. As a result, the optimal number of suppliers is small.\textsuperscript{22}

An implication of the free-entry condition (16) is that, in equilibrium, expected profits are proportional to the variance of profits. Therefore, as long as there is uncertainty, expected profits are positive and given by
\[ \mathbb{E}(\pi) = \left(\frac{\beta}{2}\right) \text{var}(\pi) = F + \left[ \left(1 - \frac{\rho}{\rho} \right) \frac{F}{f} \right]^\frac{1}{2}. \] (18)

This equation shows that equilibrium expected profits are increasing in fixed production costs. This happens because an increase in those variables increases equilibrium firm size and, as a result, the level of risk each firm faces in equilibrium. As shown in the free-entry condition (16), an increase in expected profits is required to compensate for the increase in risk. Equation (18) provides an expression for equilibrium expected profits. However, because each final good firm receives a different vector of shocks, realized profits will be distributed

\textsuperscript{22}The fact that the variance of productivity shock has no impact on the optimal number of suppliers per firm is likely the result of the specific functional forms used in the model and may not hold in other cases.
around expected profits. Final good firms with relatively more productive suppliers will produce and sell more output and, as a result, will be more profitable.

The results presented so far show that the optimal number of suppliers and the equilibrium expected profits are independent of the manager’s marginal rate of substitution between expected profits and risk, $\beta$. However, as can be seen in equation (15), this is not the case for the optimal number of production workers per supplier. Therefore, to make progress, I need to characterize the marginal rate of substitution $\beta$. Several specifications are possible. An obvious choice is to assume that $\beta$ is constant. Another possibility is to assume that $\beta$ is inversely proportional to final good price such that $\beta p^2 = \eta$. This specification of $\beta$ has two attractive features. First, it makes the distribution of employment across suppliers invariant to nominal variables

$$l^* = \left[ \frac{2f}{\eta(1-\rho)\sigma^2} \right]^{\frac{1}{2}},$$

and simplifies the characterization of the equilibrium.\textsuperscript{23} Second, as shown in Appendix A.1 at the end of the paper, the optimal distribution of employment across suppliers under productivity uncertainty becomes equivalent to the optimal distribution of input demand across suppliers under cost uncertainty. I therefore adopt that specification for the main results. In Appendix A.2 at the end of the paper, I outline the case of constant $\beta$ and show that the main characteristics of the equilibrium are not affected by this choice.

Substituting with the equilibrium number of suppliers, $n^*$ defined in (17), and the equilibrium number of production workers per supplier, $l^*$ defined in (19), into the free-entry condition (16) yields the free entry equilibrium price

$$p^* = \frac{1}{\mu} \left( 1 + (2\eta \sigma^2)^{\frac{1}{2}} \left\{ \alpha \cdot \left( (1-\rho)\sigma \right)^{\frac{1}{2}} + (\rho F)^{\frac{1}{2}} \right\} \right).$$

In equilibrium, final good firms charge an additive markup over the (expected) marginal costs of producing a unit of materials, $1/\mu$. The markup is an increasing function of the fixed production costs, the variance and correlation of production costs, and the risk aversion parameter. When fixed costs are large, average production costs are also large and, as a result, prices are higher. When there is a considerable uncertainty, firms charge a higher markup to generate greater (expected) profits to compensate for the risk associated with production.

\textsuperscript{23}In the same spirit, Wolak and Kolstad (1991) assume that the marginal rate of substitution between expected profits and risk is invariant to scale.
3.4 Market clearing

In this section, I characterize the equilibrium in the labor and final goods markets. I begin with the labor market. In equilibrium, the total number of workers associated with each final good firm is given by

\[ \Gamma^* = F + \left( \left( \frac{1 - \rho}{\rho} \right) FF \right)^{\frac{1}{2}} + \left( \frac{2F}{\eta \rho \sigma^2} \right)^{\frac{1}{2}}. \] (21)

The first term captures the labor associated with the firm’s fixed production costs, the second accounts for the labor used to pay suppliers’ fixed production costs, while the third term represents labor used in the production of materials. This equation makes clear that labor demand associated with each final good firm is increasing in fixed costs but decreasing in risk.

Each consumer is endowed with one unit of labor. Because consumers have no taste for leisure, they supply their unit of labor to the market at the prevailing wage rate. This implies that the supply of labor is equal to the mass of consumers in the economy, denoted by \( L \). The demand for labor is given by the product of the optimal labor demand per firm (\( \Gamma^* \) defined in equation (21)) and the mass of firms in the economy, denoted \( N \). It follows from the labor-market-clearing condition that the equilibrium mass of firms is given by

\[ N^* = \frac{L}{\Gamma^*}. \] (22)

This equation shows that an increase in the size of the labor force \( L \) increases the equilibrium mass of final good firms. Further, because total labor per firm (\( \Gamma^* \)) is decreasing in uncertainty, the mass of final good firms is increasing in uncertainty.\(^{24}\)

Each firm purchases materials from a disjoint set of suppliers, such that the equilibrium number of suppliers in the economy, denoted \( S^* \), is given by the product of the equilibrium mass of final good firms, defined in equation (22), and the optimal number of suppliers per firm, \( n^* \) defined in equation (17). It follows that

\[ S^* \equiv N^* n^* = \left( \frac{L}{\Gamma^*} \right) \left[ \left( \frac{1 - \rho}{\rho} \right) FF \right]^{\frac{1}{2}}, \] (23)

where \( \Gamma^* \) is defined in (21). This result shows that the total number of suppliers is increasing in the size of the market (\( L \)) and, through \( \Gamma^* \), in risk aversion (\( \eta \)) and uncertainty (\( \sigma^2 \)).

\(^{24}\)Because there is perfect competition in final goods market, I assume that the parameters of the model are such that there is a large number of firms in equilibrium.
3.5 Aggregate fluctuations

As seen in equations (13), (15), (16) and (18), the equilibrium values for the number of suppliers per firms, the number of workers per suppliers, the output price, and the mass of final good firms depend only on the moments of the shocks distribution. Because the distribution is time invariant, these aggregate variables remain constant over time. Conversely, aggregate output and expenditures depend on the realization of the productivity shocks and, as a result, fluctuate over time. To see this, first denote average worker productivity as

\[
\bar{\mu} \equiv \frac{1}{N} \int_{i \in N} \mu(i) \, di,
\]  

(24)

where \( i \) indexes final good firms and \( \mu(i) \equiv n^{-1} \sum_{k=1}^{n} z_k(i) \) denotes the average productivity of the firm \( i \)'s suppliers. Using this result, it is then possible to express aggregate output in the final goods sector as the product of mass of firms and average output per firm as follows: \( Q = N \bar{\mu} l \). By definition, when an economy is in a good state supplier productivity is higher on average then expected productivity, such that \( \bar{\mu} > \mu \). As a result, aggregate output is also higher then expected.

Because consumers spend all their income on final goods, aggregate expenditure is equal to income in equilibrium. There are two sources of income in the economy, payroll and return on equity, so that \( E = L + N \bar{\pi} \), where \( \bar{\pi} \) is average realized profits. From equation (5), realized profits for firm \( i \) are given by the difference between revenue and labor costs such that \( \pi(\mu(i)) = p_n \mu(i) l - \Gamma \). It follows that average realized profits is given by

\[
\pi(\bar{\mu}) = \frac{1}{N} \int_{i \in N} \pi(\mu(i)) \, di = p^* n^* \bar{\mu} l^* - \Gamma^*.
\]  

(25)

This equation makes clear that profits and, as a result, income fluctuate in proportion to realized production shocks. These results imply that, when an economy is in a good state income per worker increases above expectation just enough to allow consumers to buy the extra output they produced.

3.6 Welfare

Aggregate welfare in the model depends on the welfare of managers and consumers. I define both components in turn beginning with managers. The average welfare of managers, denoted \( W_M \), is defined as the utility of average profits. From equation (18), the impact of the risk born by managers on utility is equal to expected profits. This implies that the average manager’s welfare as a function of the realized aggregate shock \( \bar{\mu} \) is given by

\[
W_M(\bar{\mu}) = \pi(\bar{\mu}) - E(\pi) = \pi(\bar{\mu}) - \pi(\mu) = p^* n^* l^*(\bar{\mu} - \mu).
\]  

(26)
The second equality follows the definition of expected profits and the third from equation (25). When the aggregate shock is positive (i.e., when \( \bar{\mu} > \mu \)), firm profits are on average greater than expected and more than compensate for the risk born by managers. In that case, the utility of profits is greater than zero and managers’ welfare is positive on average. Conversely, when \( \bar{\mu} < \mu \), firm profits are on average lower than expected and managers’ welfare is negative on average.

I now turn to the consumer’s welfare, defined as real income per worker and denoted \( W_C \). Using this definition, it follows that

\[
W_C(\mu) \equiv \frac{E}{\rho^*} = \frac{n^* \bar{\mu} l^*}{\Gamma^*} = \frac{\bar{\mu}}{1 + \left( \frac{\eta \sigma^2}{2} \right)^{\frac{1}{2}} \left\{ \left[ (1 - \rho) f \right]^{\frac{1}{2}} + (\rho F)^{\frac{1}{2}} \right\}}.
\]

The first equality shows that, intuitively, real income per worker is equal to the ratio of (average) firm-level output to total number of workers per firm. The second equality uses the definitions for the equilibrium number of production workers per suppliers, number of suppliers, and total labor per firm given in equations (15), (17) and (21), respectively. Equation (27) shows that consumers’ welfare is increasing in the realized average productivity of workers, \( \bar{\mu} \).

Equation (26) and (27) make clear that time series changes in average productivity (\( \bar{\mu} \)) lead to variation in realized welfare. Because I am interested in comparing “long-run” welfare across equilibrium, I ignore the impact of transitory shocks to aggregate productivity and use expected welfare as a basis for comparison. While aggregate shocks induce variation in managers’ welfare over time, the expected managers’ welfare is always equal to zero (i.e., \( E(W_M) = 0 \)). As a result, aggregate welfare can be defined simply as expected consumers’ welfare which, from equation (27), is given by

\[
W^* = \frac{\mu}{1 + \left( \frac{\eta \sigma^2}{2} \right)^{\frac{1}{2}} \left\{ \left[ (1 - \rho) f \right]^{\frac{1}{2}} + (\rho F)^{\frac{1}{2}} \right\}}.
\]

This completes the characterization of the closed economy equilibrium.

4 The Impact of Uncertainty

The key parameter of the model described in the previous section is the variance of supplier productivity shocks, \( \sigma^2 \). Changes in this parameter can be interpreted as a change in the degree of uncertainty in the industry. In this section, I study the impacts of changes
in uncertainty on equilibrium industry characteristics and welfare. The main effects are summarized in the following proposition

**Proposition 1.** A decrease in uncertainty (i.e., a lower dispersion of productivity, \(\sigma^2\))

(a) increases the size of suppliers, but has no impact on the number of suppliers per firm.

(b) increases employment per firms and decreases the mass of final good firms.

(c) decreases the price of final goods.

(d) increases expected welfare.

**Proof.** See Appendix B.1. □

Overall, these results are intuitive. A decrease in uncertainty reduce the managers’ incentive to diversify. As shown in proposition 1, this leads to an increase in the equilibrium size of suppliers. The number of suppliers per firms is unchanged, however. This happens because a change in uncertainty has two opposite effects on the optimal number of suppliers. On the one hand, a reduction in uncertainty decreases the optimal number of suppliers conditional on firm size. On the other hand, a reduction in uncertainty increases the optimal size of each firm which increases the number of suppliers per firm. In equilibrium, these two effects offset each other exactly, such that the increase in input demand operates at the intensive supplier margin only.

As can be seen from equation (28), when there is no uncertainty, welfare is equal to the (expected) marginal productivity of labor (i.e., \(\mathbb{E}(W) \rightarrow \mu\) as \(\sigma^2 \rightarrow 0\)).\(^{25}\) This happens because as uncertainty decreases firms become larger and average production costs converge to marginal costs (see (19) and (22)). This general equilibrium result shows that, while consumers are indifferent to risk and managers break even in expectation, expected welfare still depends on the level of uncertainty in the industry. This result sets the stage for studying the impact of trade on welfare. Because trade may offer additional diversification opportunities to firms, it is possible that trade liberalization will reduce the equilibrium variance of profits (a measure of risk born by managers), increase firm-size and, as a result, increase in welfare.

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\(^{25}\) When there is no uncertainty, increasing returns to scale imply that there will be a single firm in equilibrium. The limiting result requires that the firm remains a price taker, which would be the case under perfectly contestable markets (e.g., Baumol et al. (1982)).
5 Free-Trade

In this section, I extend the model to allow for free trade between \( J \geq 2 \) identical countries of the type described in section 3. Because countries have access to the same technologies and there is no product differentiation, the conventional motives for trade are not operative.\(^{26}\) Nevertheless, as long as country-specific risk is imperfectly correlated across countries, trade offers risk diversification opportunities that are not available in a closed economy, such that there will be both trade and gains from trade in my model.

5.1 Optimal sourcing strategy under free trade

As in the closed economy, final good firms choose the set of suppliers to source from and the distribution of input demand across suppliers to maximize the expected utility of profits given in (10). As in Krugman (1979) and Melitz (2003), symmetry ensures that the equilibrium wage rate is the same in all countries (henceforth normalize to 1 without loss of generality). For simplicity, I assume there are no worldwide shocks and that country-specific shocks are uncorrelated so that

\[
\text{corr}(z_k, z_h) = \begin{cases} 
\rho & \text{if } k, h \in S_j, \\
0 & \text{if } k \in S_j \text{ and } h \in S - S_j,
\end{cases}
\]

where \( S_j \) denotes the set of (potential) suppliers in country \( j \) and \( S \) is the set of suppliers in the world. In the Appendix A.3 at the end of the paper, I show that allowing for across country correlation in productivity shocks does not affect the qualitative properties of the results.

Together, the symmetry of suppliers within countries, the absence trade costs, and the fact that countries are identical imply that the optimal number of suppliers is constant across countries and that employment is independent of the supplier’s location. It follows that firm profits are defined as in the closed economy; see equation (5). The variance of profits, however, becomes a function of the number of countries from which the firm buys inputs

\[
\text{var}(\pi) = \left(1 - \rho + \frac{n\rho}{J}\right) np^2 l^2 \sigma^2,
\]

\(^{26}\)There are no classic comparative advantage motives for trade because there is only one factor of production and expected production costs are the same in all countries. Economies of scale and consumers taste for variety do not provide a motive for trade as in Krugman (1979) because final goods are homogenous and demand is perfectly elastic.
where, as before, \( n \) denotes the total number of suppliers per final good firm.\(^{27}\) Equation (30) shows that, under free trade, firms can reduce the impact of country-specific risk by sourcing inputs from multiple countries (i.e., \( \partial \text{var}(\pi)/\partial J < 0 \)). The latter channel was not available in the closed economy such that firms would always face the country-specific risk in full (i.e., \( n \rho / J = n \rho \), when \( J = 1 \)).

Figure 3 depicts the variance of profits conditional on firm size \( (\tilde{\tau} = pnl) \) defined in equation (30) in \((n, \text{var}(\pi))\)-space. The figure brings out three important points. First, conditional on the number of suppliers, an increase in the number of countries reduces the variance of profits. Second, the number of suppliers required to reach a given level of variance, such as \( \nu \), is decreasing in the number of countries. In other words, the cost of insuring against risk is decreasing in the number of countries because firms can diversify away more of the country-level component of risk. Third, firms are fully insured against both idiosyncratic and country-specific shocks only when both the number of suppliers and the number of countries go to infinity. This happens because, while an infinite number of suppliers perfectly insures against idiosyncratic risk it does not attenuate the effect of country-specific risk.

The firm’s problem in the open economy is a generalization of its closed economy counterpart (defined in equation (12)) that accounts for the number of countries from which the firm sources

\[
\max_{n,m} \mathbb{E}[U(\pi)] = pm\mu l - F - nf - nl - \eta \sigma^2 l^2 n \left(1 - \rho + \frac{n \rho}{J}\right).
\]  

The free-entry equilibrium is characterized by the following first-order and free-entry conditions

\[
\frac{\partial \mathbb{E}[U(\pi)]}{\partial n} = 0 \iff pm\mu l - f - l = \eta \sigma^2 l^2 \left(1 - \rho + \frac{2 n \rho}{J}\right),
\]

\[
\frac{\partial \mathbb{E}[U(\pi)]}{\partial l} = 0 \iff pm - 1 = 2 \eta \sigma^2 l \left(1 - \rho + \frac{pm}{J}\right),
\]

\[
\mathbb{E}[U(\pi)] = 0 \iff pm\mu l - (F + nf + nl) = \eta \sigma^2 l^2 n \left(1 - \rho + \frac{pm}{J}\right).
\]

As expected, setting \( J = 1 \) yields the closed economy conditions defined in equations (13), (14), and (16).

Together, conditions (32)–(34) provide analytical expressions for the equilibrium demand per supplier, number of suppliers, expected price, profits, total employment associated with

\(^{27}\)This equation implicitly assumes that the optimal number of suppliers is greater than the number of countries in the world (i.e., \( n > J \)). For simplicity, I assume this is the case, but it would be straightforward to include a country-level fixed cost of contracting to limit the number of countries from which a firm would find it optimal to source from.
where \( l^* \) and \( n^* \) are the closed economy equilibrium values of production workers per supplier and number of suppliers defined in equations (15) and (17), respectively. Again, setting \( J = 1 \) yields the closed economy equilibrium outcomes defined in equations (15), (17), (20), (21), and (22) respectively.

### 5.2 Welfare

As in the closed economy, expected aggregate welfare in the open economy equilibrium is defined as real income. Using results from equations (35)–(38), it follows that

\[
W_{FT}^* = \frac{n_{FT}^* \mu_{l_{FT}}^*}{\Gamma_{FT}^*} = \frac{\mu}{1 + \left( \frac{\eta \sigma^2}{2} \right) \left\{ \left[ (1 - \rho) f \right]^{\frac{1}{2}} + \left( \frac{\rho F}{J} \right) \right\}}.
\]

This equation is a generalization of the closed economy expression presented in equation (28). Because every final good firm follows the same sourcing strategy, the expected welfare is the same in every country. As in the closed economy, welfare is decreasing in uncertainty. This completes the characterization of the open economy equilibrium.
5.3 Trade share

I now explore the determinants of bilateral trade flows in intermediate goods. Aggregate materials imports are given by the product of the total output of materials, $N^*_T n^*_T \overline{\mu}_T^*$, and the share of inputs sourced from foreign suppliers, $(J-1)/J$. It is straightforward to show that the equilibrium (expected) trade share, defined as the value of trade over the value of final goods, is given by

$$\theta^*_T \equiv \frac{\text{Trade Volume}}{\text{Revenue}} = \frac{J-1}{J} \frac{1}{p^*_T},$$

(41)

where $p^*_T$ is defined in equation (37). Using this result, it follows that

**Proposition 2.** Under free trade, the trade share is decreasing in uncertainty (i.e., the dispersion of production costs, $\sigma^2$).

**Proof.** See Appendix B.2 □

The finding that greater uncertainty decreases trade may appear counterintuitive at first. After all, an increase in risk should increase the incentive for firms to diversify their input demand across a large number of suppliers and increase their global footprints. In fact, this is exactly what firms do in partial equilibrium. From equation (35), an increase in uncertainty reduces the optimal demand per supplier which, all else equal, increases the number of suppliers per firm. However, general equilibrium adjustments imply that the optimal size of firms, defined in equation (38), goes down following an increase in risk. In equilibrium, the two effects cancel each other exactly, such that the optimal number of suppliers per firm remains the same as shown in equation (36). The increase in the mass of final good firms associated with the increase in uncertainty implies that a greater share of labor devoted to “non-productive” investment activities (i.e., paying fixed costs $f$ and $F$) which, ultimately, leads to a decrease in the trade share.

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28There is no trade in final goods in the free trade equilibrium. This happens because the optimal sourcing strategy of firms is the same, independent of their location, and firms distribute demand evenly across all suppliers. As a result firm location has no impact on realized average output per worker. For example, consider a world with two countries. Denote the average (realized) productivity of suppliers located in each country by $\overline{\mu}_1$ and $\overline{\mu}_2$, respectively, and assume that $\overline{\mu}_1 > \mu > \overline{\mu}_2$. In that case, firms in country 1 will receive on average more output than expected from their domestic suppliers, but less than expected from their foreign suppliers. The opposite pattern holds for firms located in country 2. Because firms purchase the same share of inputs from all of their suppliers, firms in both countries will receive on average the same quantity of inputs and produce the same quantity of output. This implies that average profits and, as a result, average income is the same in all countries, and that supply is just equal to demand in each country. The same argument holds for cases where $\overline{\mu}_1 > \overline{\mu}_2 > \mu$ or $\mu > \overline{\mu}_1 > \overline{\mu}_2$. 

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5.4 The Impact of Free Trade

In this section, I compare the closed economy equilibrium derived in section 3 with the free trade open economy equilibrium described in the previous section in order to evaluate the effect of moving from autarky to free trade on industry characteristics and welfare.

The impact of moving from autarky to free trade on the free-entry equilibrium characterization of the industry is summarized in the following proposition:

**Proposition 3.** A move from autarky to free trade

(a) has no impact on the size of suppliers.

(b) increases the total number of suppliers per firm, but decreases the number of domestic suppliers per firm.

(c) increases total employment per firms and decreases the mass of firms.

(d) decreases the price of final goods but increases expected profits.

**Proof.** See Appendix B.3

As shown in equation (30), an increase in the dispersion of suppliers across countries reduces the variance of profits conditional on output size and number of suppliers. By taking advantage of the new, more efficient diversification opportunities available to them under free trade, firms are able to increase their production and become more profitable. In the free trade equilibrium, the increase in the variance of profits associated with the increase in firm size is greater than the reduction in the variance in profits associated with the international sourcing of inputs. Therefore, an increase in profits is necessary to compensate firms for the additional risk they bear in the new equilibrium.

As reported in proposition 3, a move from autarky to free trade increases output per worker. This is the result of two opposing effects on average production costs. On the one hand, firms source from a greater number of suppliers which increases total fixed costs associated with the production of each final good firms. On the other hand, each firm demands a greater quantity of inputs from each of its suppliers and produces more output. Overall, the increase in output outweighs the increase in fixed production costs, such that average production costs per firm goes down following the opening to free trade. The reorganization of input demand and final good production brought about by free trade leads to increased efficiency: the share of labor devoted to fixed costs is smaller than before such that more output can be produced. It follows that free trade increases (expected) firm-level and aggregate productivity.

Comparing propositions 1 and 3 reveals that a decrease in uncertainty and a move from autarky to free trade have qualitatively equivalent impacts on the equilibrium mass and
size (measured by total employment) of final good firms. However, the underlying changes in the firms’ structure differ across types of shock. A decrease in uncertainty leads to an increase in the number of workers per supplier but has no impact on the number of suppliers per firms, whereas the opening to free trade increases the number of suppliers per firm but has no effect on the size of each supplier. Therefore, while both shocks affect the mass and the size of final good firms in the same way, they work through different suppliers margin: uncertainty affects the intensive margin of sourcing whereas free trade affects the extensive margin of sourcing.

So far, the results presented in this section show that there is trade even in the absence of traditional motives for trade. I now demonstrate that there are also gains from trade in the model.

**Proposition 4.** A move from autarky to free trade increases welfare and the gains from trade are increasing in the number of countries.

*Proof.* See Appendix B.4

Trade increases welfare because the cost of insuring against risk is lower in the open economy; conditional on the number of suppliers, firms reduce their exposure to country-specific risk when they source inputs from multiple countries. Because there are no comparative advantage or product-differentiation motives for trade in the model, the model highlights a new source of gains from trade: risk diversification.

### 6 Costly Trade

The analysis of the previous section evaluates the impact of moving from one extreme of the trade regime spectrum, autarky, to the other, free trade. In reality, most economies fall somewhere in between these two extreme cases and trade liberalization generally consists of reductions in trade barriers (not a complete removal). In this section, I extend the model to include trade costs and study the impact of a reduction in trade costs on the optimal sourcing strategy.

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29I note that, while economies of scale do not drive trade flows, they are necessary to generate gains from trade. Without them, equilibrium prices and, as a result, welfare would remain unchanged. In a sense, returns to scale play a role similar to product differentiation in new trade models with CES preferences and representative firms (e.g., Krugman (1980)). While economies of scales generate an incentive for trade in those models, taste for varieties are necessary for welfare gains because firm-level prices and output do not change following trade liberalization.
6.1 Optimal sourcing strategy under costly trade

I assume that there are variable transport costs associated with importing materials from foreign suppliers. For simplicity, the trade costs take the iceberg form such that if \( \tau \) units are shipped from a foreign country, only one unit arrives at the domestic country. The presence of trade costs implies that the optimal demand will vary depending on the location of suppliers (i.e., domestic vs. foreign) as will the optimal number of suppliers per country. So I need to distinguish between domestic and foreign values. I let \( n_D \) denote the number of domestic suppliers and \( n_X \) the number of suppliers in each foreign country. Country symmetry implies that, if firms import materials, they will have the same number of suppliers in each country. Therefore, the total number of suppliers per firm is given by 

\[ n = n_D + (J - 1)n_X. \]

As before, firms choose the number of suppliers in each country as well as the distribution of employment across suppliers in order to maximize the expected utility of profits. The firm’s problem under costly trade can be expressed as

\[
\max_{n_D, l_D, n_X, l_X} E[U(\pi)] = p\mu \left[ n_D l_D + (J - 1)n_X l_X \right] - \sum F - n_D (f + l_D) - (J - 1)n_X (f + \tau l_X) - \frac{\eta}{2} \sigma^2 \left[ l_D^2 n_D (1 - \rho + \rho n_D) + (J - 1)\tau^2 l_X^2 n_X (1 - \rho + \rho n_X) \right].
\]

As seen from the first line, revenue per unit is independent of the input’s origin. However, as seen in the second line, marginal costs are higher when materials are purchased from foreign suppliers. This happens because additional transport costs must be paid to import materials. The third line of the problem shows the impact of the variance of profits on expected utility. The key point here is that the variance of profits is increasing faster in the number and size of foreign suppliers. Again, this happens because of the trade costs.

The four first-order conditions for the costly trade problem defined in equation (42) are

\[
\frac{\partial E[U(\pi)]}{\partial n_D} = 0 \iff (p\mu - 1)l_D - f = (\eta/2)\sigma^2 l_D^2 (1 - \rho + 2\rho n_D), \tag{43}
\]

\[
\frac{\partial E[U(\pi)]}{\partial n_X} = 0 \iff (p\mu - \tau)l_X - f = (\eta/2)\sigma^2 \tau^2 l_X^2 (1 - \rho + 2\rho n_X), \tag{44}
\]

\[
\frac{\partial E[U(\pi)]}{\partial l_D} = 0 \iff \quad p\mu - 1 = \eta\sigma^2 l_D (1 - \rho + \rho n_D), \tag{45}
\]

\[
\frac{\partial E[U(\pi)]}{\partial l_X} = 0 \iff \quad p\mu - \tau = \eta\sigma^2 \tau^2 l_X (1 - \rho + \rho n_X). \tag{46}
\]

I note that the number of countries \( J \) does not appear in those equations because I defined the objective function in terms of the country-level number of suppliers, \( n_D \) and \( n_X \), instead of the total number of suppliers, \( n \). The conditions have interpretations similar as in the
closed economy and free trade equilibrium and, together, define the efficient scale of final good firms as the combination of suppliers and demand per supplier such that average costs are at their lowest.

Combining the first order conditions for the number of domestic suppliers and for the demand per domestic suppliers defined in equations (43) and (45), respectively, provides an analytical expression for the optimal demand per domestic suppliers under costly trade, \( l^*_D \). Similarly, combining the two “foreign” first order conditions (44) and (46) provides an analytical expression for the optimal demand per foreign supplier, \( l^*_X \). These optimal demands can be expressed as

\[
l^*_D = l^* \quad \text{and} \quad l^*_X = l^*/\tau, \tag{47}\]

where \( l^* \) is the optimal demand per supplier defined in equation (15).

Analytical expressions for the optimal number of suppliers in the costly trade equilibrium are not tractable. Instead, I rely on two mappings from the optimal number of domestic suppliers, \( n^*_D \), to the optimal number of foreign suppliers, \( n^*_X \), to analyze the impact of changes in the economic environment on the optimal sourcing strategy. Combining first-order conditions (45) and (46) provides the first mapping

\[
n^*_X = \lambda(n^*_D) \equiv n^*_D - \frac{\tau - 1}{\rho} \left( \frac{1 - \rho}{2nf\sigma^2} \right)^{1/2}. \tag{48}\]

This mapping is depicted in Figure 4 in \((n^*_D, n^*_X)\)-space. As shown in the figure, efficiency requires that the number of foreign suppliers be increasing in the number of domestic suppliers and that the number of foreign suppliers be smaller than the number of domestic suppliers (as long as trade costs are not trivial (i.e., \( \tau > 1 \)), in which case \( n^*_X = n^*_D \)). Intuitively, firms strike a balance between the incentive to diversify country-specific risk and the additional costs associated with purchasing foreign inputs.

The free-entry condition provides the second mapping from the optimal number of foreign suppliers to the optimal number of domestic suppliers

\[
n^*_X = \gamma(n^*_D) \equiv \left\{ \frac{1}{J - 1} \left[ \left( \frac{1 - \rho}{\rho} F \right) - n^*_D \right] \right\}^{1/2}. \tag{49}\]

As illustrated in Figure 4, this mapping shows that the free-entry condition requires that the number of foreign suppliers be decreasing in the number of domestic suppliers. This happens because an increase in the number of suppliers increases production costs. In order to leave costs unchanged, an increase in the number of domestic suppliers must be accompanied by a decrease in the number of foreign suppliers. The optimal number of suppliers in the domestic
country and in each foreign country is given by the intersection of the two mappings, \( \lambda(n_D) \) and \( \gamma(n_D) \), as indicated by point E in Figure 3.\(^{30}\)

### 6.2 Trade liberalization

I now evaluate the effects of a reduction in trade costs on the intensive and extensive margins of sourcing. The impact on the intensive margin follows directly from equation (47). As expected, because a decrease in marginal costs increases the optimal demand, the quantity demanded from each foreign supplier is greater when trade costs are lower. Equation (47) clearly shows that demand for imported materials converges to 0 as trade costs become prohibitive (i.e., \( l_X^* \rightarrow 0 \) as \( \tau \rightarrow \infty \)) and to the demand for domestic inputs as trade costs become trivial (i.e., \( l_X^* \rightarrow l_D^* \) as \( \tau \rightarrow 1 \)).

To evaluate the impact on the extensive margin, I use the two mappings defined in the previous subsection. The effects of changes in trade costs are depicted in Figure 5. First, a decrease in trade costs makes the intercept of the efficiency condition, defined in equation (48), less negative but has no impact on the slope. Second, as can be seen from equation (49), changes in trade costs have no impact on the free-entry mapping. Starting from equilibrium point A, a decrease in trade costs will lead to a decrease in the number of domestic suppliers and an increase in the number of foreign suppliers; the equilibrium moves from point A to point B.

The impact of trade costs on the optimal sourcing strategy is summarized in the following proposition

**Proposition 5.** Under costly trade, a decrease in trade costs

(a) increases the quantity of inputs demanded from each foreign supplier, but has no impact on the quantity demand from domestic suppliers.

(b) decreases the number of domestic suppliers per firm and increases the number of foreign suppliers per firm.

\(^{30}\)The presence of trade barriers introduces a motive for trade in homogeneous final goods. Because countries are symmetric, the optimal sourcing strategy and, as a result, the expected output will be the same for all final good firms independent of their location. However, because trade costs lead to differences in input demand across domestic and foreign suppliers, realized output will vary across countries. For example, consider the two country world defined in footnote 28 under the assumption that \( \overline{\mu}_1 > \mu > \overline{\mu}_2 \). In that case, firms in country 1 will receive on average more output than expected from their domestic suppliers but less than expected from their foreign suppliers. The opposite pattern holds for firms located in country 2. Because firms purchase a greater share of inputs from their domestic suppliers, firms in country 1 will produce more output compared to firms in country 2. This implies that firm profits and, as a result, welfare will be above expectations in country 1 and below expectations in country 2. In that context, consumers could ensure against country-level shocks by signing contracts that specifies world output of final goods is to be divided into equal shares in all states of the world. This would lead to international risk sharing through trade in homogeneous final goods.
Figure 4 illustrates another interesting point: there can be selection into exporting even in the absence of fixed trade costs. Intuitively, when *ad valorem* trade barriers are sufficiently large, the reduction in the variance of profits associated with sourcing from foreign suppliers is not large enough to justify the increase in input price (see equation (48)). Consider the equilibrium point C for example. At that point the optimal number of foreign suppliers is negative which, of course, is not possible. Therefore, firms buy their inputs exclusively from domestic suppliers. In a sense, the cost of contracting with suppliers \( f \) has a similar impact as the fixed export costs in heterogeneous firm model (e.g., Melitz (2003) and Helpman et al. (2008)) and leads to selection into trade.

### 6.3 Increased uncertainty

Now, I turn to the impact of changes in uncertainty on the optimal sourcing decision under costly trade. From equations (47), (48), and (49), it follows that

**Proposition 6.** Under costly trade, an increase in uncertainty

(a) decreases the quantity of inputs demanded from each supplier.

(b) decreases the number of domestic suppliers per firm and increases the number of foreign suppliers per firm.

**Proof.** See Appendix B.6. □

When there is considerable uncertainty, the reduction in the variability of profits associated with contracting with additional suppliers is substantial. In that case, firms find it optimal to pay fixed contracting costs to purchase from additional number of domestic and foreign suppliers. Because firms source from a greater number of suppliers, the quantity demanded per supplier goes down.

Equation (48) shows that, conditional on the number of domestic suppliers, a reduction in uncertainty reduces the optimal number of foreign suppliers. This happens because a decrease in uncertainty decreases the benefits associated with diversification. Eventually, there comes a point where firms do not find that the additional reduction in risk associated with contracting with foreign suppliers is worth the premium paid for imported inputs. As illustrated by point C, when uncertainty is small enough there is no trade in equilibrium.
7 Econometric Evidence

In this section, I use data on disaggregated U.S. imports to test the empirical validity of the two main predictions of the model related to firms’ sourcing decisions. First, I study the relationship between input demand and suppliers’ characteristics. Consistent with the predictions of the theoretical model, the empirical results show that U.S. firms purchase a larger fraction of their inputs from suppliers characterized with low production costs, low trade barriers, and low risk. Second, I examine the relationship between industry characteristics and the distribution of input demand across suppliers. As predicted by the model, I find that industries characterized with higher levels of risk have more dispersed input demand. The data shows that U.S. firms purchase inputs from a greater number of suppliers and spread their input demand more evenly across suppliers when there is greater uncertainty in the upstream industry.

7.1 Testable implications

The first testable implication of the model relates input demand to suppliers’ characteristics. The model predicts that the quantity of inputs delivered by each supplier to the downstream firm is given by

\[ m = zl = \left[ \frac{2f}{\eta(1-\rho)\sigma^2} \right]^{1/2} \zeta \tau. \]  

(50)

The first equality shows that the quantity of materials delivered depends on the deterministic choice of employment and the stochastic productivity of workers (\(z\)). The second equality replaces \(l\) with the optimal level of employment defined in (47). Equation (50) suggests that in a cross-section of suppliers the share of input demand purchased from a supplier should be decreasing in the associated risk (\(\nu \equiv (1-\rho)\sigma^2\)), production costs (\(c = 1/z\)) and trade barriers (\(\tau\)).

From equation (50), the optimal demand for input \(g\) per country-\(j\) supplier at time \(t\) can be expressed as

\[ \ln m_{jgt} = \theta_{gt} + \theta_1 \ln \nu_{jgt} + \theta_2 \ln c_{jgt} + \theta_3 \ln \tau_{jgt} + \epsilon_{jgt}. \]  

(51)

The first term on the right-hand side of equation (51), \(\theta_{gt}\), is a vector of industry-year dummies common to all suppliers. In terms of the model, these fixed effects control for the

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31 Gervais (2016) uses disaggregated data on U.S. imports to quantify the impact of expected input costs and variability on the distribution of input demand across countries. By contrast, in the current paper I augment the trade data with firm-level information from the World Bank Exporter Dynamic Dataset to derive supplier-level measures of input demand. Further, I explore the determinants of the extensive margin of sourcing (i.e., the number of suppliers) which is absent from Gervais (2016).

30
risk aversion of downstream firms, as well as the suppliers fixed production costs. In the 
application, they also remove the impact of industry-year differences in production costs, 
trade costs, and risk. The last term in equation (51), $e_{gt}$, is a residual that contains factors 
not included in the model that can affect demand per supplier but are uncorrelated with the 
included regressors. As stated in propositions 5 (a) and 6 (a), input demand is decreasing 
in the degree of uncertainty, the input costs, and the barriers to trade associated with the 
supplier. In other words, the model predicts that $\theta_1 < 0$, $\theta_2 < 0$, and $\theta_3 < 0$.

The second testable implication of the model relates the distribution of input demand 
across suppliers to industry-level risk characteristics. According to the theoretical model, an 
increase in uncertainty or a decrease in trade costs increases the number of foreign suppliers 
per firm. Together these results suggest that an increase in industry-level uncertainty will 
lead to a more dispersed distribution of input demand across foreign suppliers. In the 
econometric results below, I report estimates from regressions of the form

$$\ln D_{gt} = \omega_0 + \omega_1 \ln \nu_g + \omega_2 \ln \tau_g + e_{gt}$$

(52)

where $D_{gt}$ is a measure of input demand dispersion for input $g$ in year $t$, $\nu_g$ is an industry-level 
measure of risk, $\tau_g$ is the corresponding measure of trade barriers, and $e_{gt}$ is an orthogonal 
error term. In light of propositions 5 (b) and 6 (b), I expect that $\omega_1 > 0$ and $\omega_2 < 0$.

7.2 Data and measurement

I construct the variables for the empirical analysis by combining disaggregate data on U.S. 
imports of manufacturing goods from the U.S. Census Bureau and information on exporting 
firms from the World Bank’s Export Dynamics database.\footnote{The trade data is available on Peter Schott’s website at http://faculty.som.yale.edu/peterschott/ sub_international.htm. The World Bank’s Export Dynamics dataset can be downloaded from http://go.worldbank.org/DAX4OEU1Z0.}

The trade data includes both quantity and value information, such that it is possible 
to calculate proxies for import prices using unit values. While unit value information 
is available for a broad set of countries from the United Nation’s Comtrade database, 
restricting the analysis to U.S. has two important advantages. First, each observation is 
associated with a ten-digit Harmonized Trade Schedule (HTS10) code. In contrast, to 
maintain a consistent classification across countries, the Comtrade data classifies products 
using the more aggregated six-digit Harmonized System (HS6) codes.\footnote{The HTS10 codes are an extension of HS6 codes. The first six digit map of each HTS10 code maps to a single six-digits Harmonized System codes. The remaining 4 digits provides additional detail on the characteristics of the products.} Because I am 
interested in explaining multi-sourcing decisions, narrowly defined product categories are 
preferable in order to minimize within-category product differentiation which could lead to
an overestimation of multi-sourcing. Second, the U.S. import data records information on duties paid and freight costs, so it is possible to construct measures of trade costs for each observation in the sample. This information is not available in the Comtrade data.\footnote{The Comtrade data collects information on the c.i.f. import values and f.o.b. export values. It is therefore possible to obtain a measure of c.i.f. costs by taking the difference between import and export values. However, this procedure has many caveats. First, there is significant sample attrition because export values are not reported for all observations. Second, there is a lot of measurement error in export values which results is negative c.i.f. costs. Third, these measures of trade costs do not include tariff barriers.}

Estimating the intensive margin equation (51) requires data on the quantity of inputs imported from each supplier, the trade costs associated with each of these suppliers, as well as measures of expected costs and risk. I measure imports per supplier for each product-year as the number of units imported from a country divided by the number of exporting firms in that country.\footnote{For each industry-country-year category in my sample, I estimate the number of exporting firms using the “median number of exporters per destination” as reported in the Exporter Dynamics database. Because the Exporter Dynamics database collects information on multiple countries, the highest level of disaggregation available are HS6 codes. For the empirical analysis, I assume that firms classified as exporting in a given HS6 industry export all HTS10 products in that industry.} To construct \textit{ad valorem} measures of trade barriers, I first calculate the ratio of the sum of reported freight costs and duties paid to total import value, $t_{jgt}$, then I add one to these ratios such that $\hat{\tau}_{jgt} = 1 + t_{jgt}$. To obtain measures of expected costs and variance of input price, I need to formalize the stochastic shock process. For the benchmark estimation, I assume that the distribution of productivity is the same in all periods for each country-industry category such that $c_{gjt} = c_{gj} + u_{gjt}$ with $u_{gjt} \sim G(0, \sigma_{jg}^2)$. In that case, the expected level and variance of production costs are given by $E_t(c_{gjt}) = c_{gj}$ and $E_t(\text{var}(u_{gjt})) = \sigma_{jg}^2$, respectively. Under these assumptions, I can measure expected production costs for each product-country category as the average unit value over all years in the sample, and use the standard deviation of unexpected production cost shocks as a proxy for the corresponding risk measure, $\nu_{jgt}$.

To estimate the extensive margin of sourcing (equation (52)), I use the product-level averages of the measures of trade barriers and risk to proxy for $\tau_g$ and $\nu_g$, respectively.\footnote{I obtain these averages using a procedure similar to Koren and Tenreyro (2007), and Hanson et al. (2015). First, I regress the measures of risk and trade barriers on exporter, HTS10 product, and year fixed effects. Then, I estimate the mean using the point estimates on the product dummies.} I derive the benchmark measure of demand dispersion ($D_{gt}$) using information on export concentration collected by the Exporter Dynamics database. For each industry-country-year observation, the database reports an Herfindahl index, which I denote $H_{jgt}$, that characterizes the distribution of export values across firms. I combine the Herfindhal indices with measures of import demand shares ($s_{jgt}$) to obtain an industry-level Herfindhal index as follows: $H_{gt} = \sum_j s_{jgt}^2 H_{jgt}$. By definition, the Herfindhal index ranges from 0 to 1, moving from a dispersed input demand (i.e., a large number of small suppliers) to a concentrated input demand (i.e., a single supplier). Because an increase in the index is inversely proportional to
demand dispersion, I let $D_{gt}^A \equiv H_{gt}^{-1}$. The advantage of this measure of dispersion over a simple count of the number of suppliers is that it takes into account the distribution of input demand across suppliers. As such, it provides a more accurate description of the importance of multi-sourcing. Nevertheless, as a check, I also consider two simpler measures of demand dispersion. The first is the count of firms from which US firms purchase a given input, i.e., $D_{gt}^B = \sum_j n_{jgt}$, where $n_{jgt}$ denotes the number of exporting firms. The second is the count of countries from which U.S. firms purchase a given input, i.e., $D_{gt}^C = \sum_j I_{jgt}$, where $I_{jgt}$ is an indicator variable equal to 1 if U.S. firms import input $g$ from country $j$ at time $t$, and 0 otherwise. The advantage of this last measure is that it does not require firm-level information and can therefore be defined for a broader sample of countries. Appendix Table A.1 provides a list of countries included in each sample.

Panel A and B of Table 1 present summary statistics for the variables included in estimating the intensive margin equation (51) and the extensive margin equation (52), respectively. For the benchmark results, I follow the theoretical model and restrict the sample to intermediate goods. Together, intermediate goods account for more than half of the total value of U.S. import in my sample. The sample covers years 1997 to 2014.

7.3 Intensive margin of sourcing

The results from estimating equation (51) are reported in Table 2. All regressions include a full set of product-year fixed effects such that the estimates are not driven by cross-sectional or time-series variation. The first column reports OLS results from regressing input demand on the variance of production cost shocks (i.e., risk). The second column estimates the effects of differences in expected production costs and trade costs on the demand for inputs. The third column estimates the full model which includes the measures of risk, expected production costs and trade barriers. As seen in the table, all point estimates are large, negative, and statistically significant. These results suggest that, as predicted by the theoretical model, U.S. firms import a larger share of their inputs from low-costs, low-risk suppliers.

A potential concern with the estimation results is the presence of correlated measurement error on both sides of the equation. This happens because both import volumes and production costs are computed from quantities. However, because expected production costs are averages over time the problem is likely to be less important than if quantities and costs were contemporaneous. Nevertheless, as a check, I re-estimated the model using lag of unit values as instruments for expected production costs. As can be seen in column (4) of Table

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37The most disaggregated classification available in the Exporter Dynamics database is the HS6. For the empirical analysis, I assume that all HTS10 products within a country-year-HS6 category are characterized by the same Herfindahl index. Nevertheless, the measure of demand dispersion varies at the HTS10-year level because of differences across HTS10 in the set of countries from which the U.S. imports.
2, using 2SLS has little impact on the point estimates. They remain large, negative, and statistically significant as expected.

The advantage of using firm information to obtain a proxy for supplier-level input demand is that the empirical model follows the theoretical model closely. A disadvantage, however, is that the sample is restricted to observations for which firm-level data is available. To increase coverage, I reestimate the model using input demand defined at the country-level, essentially assuming that there is a single supplier in each country. The OLS results are presented in column (5) of Table 2. As seen in the table, the estimated coefficients are negative and statistically significant, and the magnitudes are similar to the benchmark results.

The estimation results are robust to a number of assumptions. First, suppose that production costs follow a random walk instead of being drawn from a time invariant distribution. In that case, \( c_{gj,t} = c_{gj,t-1} + u_{gj,t} \) such that the expected level and variance of production costs are given by \( E_t(c_{gj,t}) = c_{gj,t-1} \) and \( E_t(\text{var}(u_{gj,t})) = \sigma^2_{gj,t} \), respectively. This suggests that expected production costs should be measured as lag unit values. However, the measures of risk remain the same as before because the impact of time is captured by the industry-year fixed effects. Second, while the theoretical model is confined to intermediate goods, it is interesting to estimate the model in a broader subset that includes all types of manufactured products. Finally, while HTS10 product categories are quite narrow, it is possible that there is still product differentiation at the unobserved HTS12 level. If this is the case, I may overestimate the extent of multi-sourcing. To minimize the impact of product differentiation, I reestimate the model using a narrower sample restricted to homogeneous intermediate goods (as defined in Rauch (1999)). The robustness results are reported in Table 3. For convenience, the table also reports the benchmark results. As seen in the table, the point estimates are large, negative and statistically significant in all cases.

### 7.4 Extensive margin of sourcing

The results from estimating equation (52) are presented in Table 4.\(^{38}\) Each panel reports the results for a different measure of input demand dispersion. Panel A uses the inverse Herfindhal, while panels B and C use, respectively, the total number of suppliers and the total number of countries from which U.S. firms buy a given input. For each dependent variable, I report results from three specifications. In the first and second column I estimate, in turn, the impact of risk and trade barriers on the extensive margin of sourcing. In the third column, I report the results from a richer model that includes both the measures of risk and the measures of trade barriers. Consistent with the theoretical model, the results reported in Table 4 suggest that multi-sourcing is more prevalent for products characterized

\(^{38}\)The table presents (product) random effects estimates. I also estimated the model using OLS. The results are very similar overall, so I do not report them to conserve space.
by high levels of risk and low trade costs. As seen in the table, the point estimates for the measures of risk are all positive and statistically significant, whereas the point estimates for trade barriers are all negative and statistically significant.

I evaluate the robustness of these estimates to a number of assumptions. The results are presented in Table 5. For convenience, the benchmark results are reproduced in the first column of the table. As seen in the table, the positive association between the measures of industry risk and input demand dispersion is quite robust. Assuming production costs follow a random walk, increasing the sample to include all manufacturing products, or restricting the sample to only homogenous intermediate goods has little impact on the point estimates. Conversely, the estimated coefficients on trade costs vary across dependent variables and sample definition. Out of eight point estimates, two are negative and significant, one is negative but insignificant at conventional levels, two are positive and insignificant, and four are positive and statistically significant.

Overall, the results presented in this section provide empirical support to two of the main predictions of the theoretical model. First, within a narrowly defined product category, U.S. firms seem to buy more inputs from low-risk suppliers. Second, U.S. firms purchase inputs characterized with high levels of risk from a greater number of suppliers.

8 Conclusion

The analysis in this paper shows that management of supply-chain risk provides an independent motive for trade and highlights a new channel through which trade can increase welfare. To focus the analysis on the role of risk and make matters as simple as possible, these results were derived under strict assumptions. A natural next step is to explore the extent to which these results generalize. While a formal analysis is well beyond the scope of this paper, it is interesting to speculate about the implications of one of the key potential extensions.

An obvious way to generalize the model would be to allow for firm heterogeneity. In the representative firm model developed in this paper, as in new trade models such as Krugman (1979) and Krugman (1980), all firms trade. However, it is well known that only a small fraction of firms trade and that these firms are more productive (e.g., Bernard and Jensen (1995)). As a result, current workhorse models of international trade feature firms that are heterogeneous in productivity and selection into trade (e.g., Eaton and Kortum (2002) and Melitz (2003)). Incorporating these features into my model of sourcing decisions would provide interesting firm-level predictions. For example, I would expect the more productive firms to be larger and source from a broader set of countries. Importantly, however, it should not change the main results of the model with regards to the effect of uncertainty on trade and welfare.
References


Brennan, M. J. (1975). The optimal number of securities in a risky asset portfolio when there are fixed costs of transacting: Theory and some empirical results. *Journal of Financial and Quantitative Analysis* 10(03), 483–496.


Figure 1: Distribution of U.S. imports
Labor costs only
Labor costs and uncertainty
Efficient scale
ACp*1/\mu_{n,l}

Figure 2: Average costs curve
Figure 3: Conditional variance of profits
Figure 4: Optimal number of domestic and foreign suppliers
Figure 5: Changes in trade costs and uncertainty
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*Notes:* The table presents summary statistics for the two main variables used in the empirical analysis. All variables are in logs.
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<td>−1.23*</td>
<td>−1.26*</td>
<td>−1.20*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.03)</td>
<td>(0.00)</td>
<td></td>
</tr>
<tr>
<td>Log trade costs</td>
<td>−5.92*</td>
<td>−5.53*</td>
<td>−6.08*</td>
<td>−5.32*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.10)</td>
<td>(0.16)</td>
<td>(0.05)</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.35</td>
<td>0.46</td>
<td>0.50</td>
<td>0.52</td>
<td>0.46</td>
</tr>
<tr>
<td>Nb. of obs.</td>
<td>109,858</td>
<td>109,858</td>
<td>109,858</td>
<td>78,778</td>
<td>1,220,715</td>
</tr>
<tr>
<td>Estimator</td>
<td>OLS</td>
<td>OLS</td>
<td>OLS</td>
<td>IV</td>
<td>OLS</td>
</tr>
</tbody>
</table>

**Notes:** The table presents OLS results from regressing U.S. import demand on supplier-year characteristics. See main text for variable definitions. All regressions contain a full set of industry-year fixed effects. Robust standard errors in parentheses; * and ** indicate 5 and 1 percent significance levels, respectively. An observation is a product-supplier-year combination. The sample is restricted to intermediate goods and covers the period from 1997 to 2014.
## TABLE 3
### Robustness: Import demand and suppliers’ characteristics

<table>
<thead>
<tr>
<th></th>
<th>Benchmark results</th>
<th>Random walk products</th>
<th>All manuf. intermediates</th>
<th>Homogeneous intermediates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log variance of shocks</td>
<td>−0.60** (0.01)</td>
<td>−0.91** (0.01)</td>
<td>−0.72** (0.01)</td>
<td>−0.58** (0.02)</td>
</tr>
<tr>
<td>Log expected prod. costs</td>
<td>−1.23** (0.01)</td>
<td>−0.51** (0.02)</td>
<td>−1.09** (0.01)</td>
<td>−1.40** (0.03)</td>
</tr>
<tr>
<td>Log trade costs</td>
<td>−5.53** (0.10)</td>
<td>−5.30** (0.16)</td>
<td>−6.18** (0.07)</td>
<td>−5.26** (0.18)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.50</td>
<td>0.46</td>
<td>0.53</td>
<td>0.44</td>
</tr>
<tr>
<td>Nb. of obs.</td>
<td>109,858</td>
<td>76,454</td>
<td>271,586</td>
<td>37,228</td>
</tr>
</tbody>
</table>

**Notes:** The table presents OLS results from regressing U.S. import demand on supplier-year characteristics. See main text for variable definitions. All regressions contain a full set of industry-year fixed effects. Robust standard errors in parentheses; * and ** indicate 5 and 1 percent significance levels, respectively. An observation is a product-supplier-year combination. The sample covers the period from 1997 to 2014.
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Inverse Herfindahl</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log variance of shocks</td>
<td>$0.26^{**}$</td>
<td>$0.25^{**}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>Log trade costs</td>
<td>$-3.13^{**}$</td>
<td>$-0.84^{**}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.33)</td>
<td>(0.34)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.09</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>Nb. of obs.</td>
<td>51,955</td>
<td>51,955</td>
<td>51,955</td>
</tr>
<tr>
<td><strong>Panel B: Number of suppliers</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log variance of shocks</td>
<td>$0.02^{**}$</td>
<td>$0.02^{**}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td></td>
</tr>
<tr>
<td>Log trade costs</td>
<td>$-0.37^{**}$</td>
<td>$-0.16^{**}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.05)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Nb. of obs.</td>
<td>51,955</td>
<td>51,955</td>
<td>51,955</td>
</tr>
<tr>
<td><strong>Panel C: Number of countries</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log variance of shocks</td>
<td>$0.03^{**}$</td>
<td>$0.02^{**}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td></td>
</tr>
<tr>
<td>Log trade costs</td>
<td>$-0.76^{**}$</td>
<td>$-0.54^{**}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

**Notes:** The table presents results from regressing measures of import demand dispersion on product-level measures of risk and trade costs. All regressions include product-level random effects. Robust standard errors in parentheses; * and ** indicate 5 and 1 percent significance levels, respectively. An observation is a product-year combination. The sample is restricted to intermediate goods and covers the period from 1997 to 2014.
### TABLE 5
**Robustness: Multi-sourcing and industry characteristics**

<table>
<thead>
<tr>
<th></th>
<th>Benchmark results</th>
<th>Random walk</th>
<th>Full sample</th>
<th>Homogeneous intermediates</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Inverse Herfindahl</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log variance of shocks</td>
<td>0.25**</td>
<td>0.25**</td>
<td>0.14**</td>
<td>0.33**</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Log trade costs</td>
<td>-0.84**</td>
<td>-0.61*</td>
<td>-2.83**</td>
<td>8.11**</td>
</tr>
<tr>
<td></td>
<td>(0.34)</td>
<td>(0.35)</td>
<td>(0.25)</td>
<td>(0.60)</td>
</tr>
<tr>
<td>R²</td>
<td>0.09</td>
<td>0.08</td>
<td>0.09</td>
<td>0.07</td>
</tr>
<tr>
<td>Nb. of obs.</td>
<td>51,955</td>
<td>48,605</td>
<td>95,270</td>
<td>20,113</td>
</tr>
<tr>
<td><strong>Panel B: Number of suppliers</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log variance of shocks</td>
<td>0.02**</td>
<td>0.04**</td>
<td>0.05**</td>
<td>0.01**</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Log trade costs</td>
<td>-0.16**</td>
<td>-0.03</td>
<td>2.25**</td>
<td>0.82**</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.06)</td>
<td>(0.05)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>R²</td>
<td>0.06</td>
<td>0.06</td>
<td>0.09</td>
<td>0.05</td>
</tr>
<tr>
<td>Nb. of obs.</td>
<td>51,955</td>
<td>48,605</td>
<td>95,270</td>
<td>20,113</td>
</tr>
<tr>
<td><strong>Panel C: Number of countries</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log variance of shocks</td>
<td>0.02**</td>
<td>0.05**</td>
<td>0.02**</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Log trade costs</td>
<td>-0.54**</td>
<td>0.04</td>
<td>0.38</td>
<td>0.70**</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.03)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>R²</td>
<td>0.01</td>
<td>0.03</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Nb. of obs.</td>
<td>108,195</td>
<td>92,376</td>
<td>187,213</td>
<td>48,527</td>
</tr>
</tbody>
</table>

**Notes:** The table presents results from regressing measures of import demand dispersion on product-level measures of risk and trade costs. All regressions include product-level random effects. Robust standard errors in parentheses; * and ** indicate 5 and 1 percent significance levels, respectively. An observation is a product-year combination. The sample covers the period from 1997 to 2014.
A Technical Appendix A

A.1 Costs uncertainty model

Suppose that instead of contracting on the expected amount of inputs, i.e., the number of workers each suppliers should employ, final good firms contracted on the actual amount of materials to be delivered. In this case, firm profits are given by

$$\pi = pq - w \left( F + nf + \sum_{k=1}^{n} c_k m_k \right),$$  \hspace{1cm} (A.1)

where $n$ denotes the (endogenous) number of suppliers from which the firm buys inputs. Note that the uncertainty now shows up in the suppliers’ production costs instead of the final good firms’ output. This happens because the number of units delivered by each supplier, $m_k$, is known with certainty, but the cost of these units, $c_k$, depends on worker’s productivity which is unknown ex ante.

Managers maximize the expected utility of profits by choosing how much output to produce conditional on the market price. As before, this involves two interrelated decisions. First, the manager chooses the set of suppliers to contract with. Second, he chooses the allocation of input demand across the selected suppliers. Because suppliers are identical ex ante (i.e., at the moment contracts are signed), the optimal share of demand is constant across suppliers and given by $1/n$. It follows that output is given by the product of the number of suppliers and the quantity of inputs demanded from each supplier, $q = nm$, where $m$ denotes demand per supplier.

Similar to the benchmark model, I assume that the expected costs, the variance of costs, and the correlation between the production costs are common across suppliers and respectively given by

$$\mathbb{E}(c_k) = \mu, \quad \text{var}(c_k) = \sigma^2, \quad \text{and} \quad \text{corr}(c_k, c_h) = \rho \in (0, 1), \quad \forall \, k, h \in S, \ k \neq h. \hspace{1cm} (A.2)$$

Using these assumptions and equation (A.1), it follows that the variance of profits can be expressed as

$$\text{var}(\pi) = \text{var} \left( \sum_{s=1}^{n} c_s m_s \right) = \left( \frac{1 - \rho}{n} + \rho \right) q^2 \sigma^2. \hspace{1cm} (A.3)$$

Substituting with this result into the expected utility function (10), the firm’s problem can be expressed in terms of choosing the number of suppliers and the quantity of inputs purchased from each supplier as follows

$$\max_{n,m} \mathbb{E} [U(\pi)] = pnm - (F + nf + n\mu m) - (\beta/2) \left( 1 - \rho + n\rho \right) \sigma^2 m^2 n. \hspace{1cm} (A.4)$$

The two first-order conditions for the firm’s problem defined in (A.4) are

$$\frac{\partial \mathbb{E} [U(\pi)]}{\partial n} = 0 \iff (p - \mu) m - f = (\beta/2) \left( 1 - \rho + 2n\rho \right) \sigma^2 m^2, \hspace{1cm} (A.5)$$
\[
\frac{\partial E[U(\pi)]}{\partial m} = 0 \quad \Leftrightarrow \quad p - \mu = \beta (1 - \rho + n\rho) \sigma^2 m. \tag{A.6}
\]

Equation (A.5) states that, conditional on demand per supplier \((m)\), the marginal revenue from contracting with an additional supplier must be just equal to the marginal increase in risk associated with adding an extra supplier (and increasing output by \(m\)). Equation (A.6) states that the marginal revenue from increasing the quantity of inputs demanded from each supplier must be equal to the corresponding marginal increase in risk. Together, conditions (A.5) and (A.6) imply that firms operate at the efficient scale.

Combining the two first-order conditions (A.5) and (A.6) provides an analytical expression for the optimal demand per supplier

\[
m^* = \left[ \frac{2f}{\beta(1-\rho)\sigma^2} \right]^{\frac{1}{2}} = \frac{l^*}{p}, \tag{A.7}
\]

where \(l^*\) is employment per worker defined in equation (19). This results shows that when the manager’s marginal rate of substitution between expected profits and risk in the input uncertainty model is inversely proportional to final good price such that \(\beta p^2 = \eta\), the optimal number of workers per suppliers is equal to the optimal input demand per supplier in the cost uncertainty model.

### A.2 Constant risk aversion parameter

In this appendix, I explore the properties of the model when the managers’ marginal rate of substitution between expected profits and risk is constant. In that case, the optimal number of workers per supplier is given by equation (15) and can be expressed as

\[
l^*_\beta = \frac{l^*}{p^*}. \tag{A.8}
\]

where \(l^*_\beta\) and \(p^*_\beta\) denote, respectively, the equilibrium number of workers and price of a unit of final good when the marginal rate of substitution between expected profits and risk is constant and given by \(\beta\). Those values are distinct from the equilibrium number of workers per suppliers \((l^*\) defined in equation (15) and price \((p^*\) defined in equation (20)) when \(\beta p^2 = \eta\). An unappealing implication of equation (A.8) is that the distribution of employment depends on the nominal price. This implies inflation or changes in units of measurement for currency would affect the distribution of employment, which does not seem very realistic.

Substituting with the optimal number of workers defined in (A.8) into the free entry condition (16), it is straightforward to show that the optimal number of suppliers per firm is still given by equation (17). Combining the results obtained so far, it is possible to solve for the equilibrium price as a function of the parameters of the model

\[
p^*_\beta = \frac{1}{\mu - (2\beta\sigma^2)^{1/2} \left\{ [(1-\rho)f]^{1/2} + (\rho F)^{1/2} \right\}}. \tag{A.9}
\]

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Prices will be positive if and only if productivity is large enough relative to the costs and risk parameter, i.e., when \( \mu > (2\beta \sigma^2)^{1/2} \left\{ \left[ (1 - \rho) f \right]^{1/2} + (\rho F)^{1/2} \right\} \). While the expression for equilibrium price is different from the benchmark model, its qualitative properties are the same. The equilibrium price is increasing in fixed costs (\( F \) and \( f \)) and risk related parameters (\( \beta \) and \( \sigma^2 \)). Substituting with equilibrium price defined in equation (A.9) in equation (A.8) yields an analytical expression for the optimal number of workers per supplier which depends only on parameters of the model.

Substituting with the optimal number of workers per suppliers, number of suppliers and equilibrium price into the definition of profits given in equation (5), it follows that equilibrium expected profits in the case of constant \( \beta \) can be expressed as

\[
\pi^*_{\beta} = \frac{\pi^*}{p^*_{\beta}}.
\] (A.10)

where \( \pi^* \) is the equilibrium profit in the benchmark model. Finally, the total number of workers associated with each final good firm is given by

\[
\Gamma^*_{\beta} = F + \left\{ \left[ \left( \frac{1 - \rho}{\rho} \right) f F \right]^{1/2} + \left( \frac{2F}{\rho \sigma^2} \right)^{1/2} \right\} \frac{1}{p^*_{\beta}}.
\] (A.11)

It follows from the labor-market-clearing condition that the equilibrium mass of firms is given by

\[
N^*_{\beta} = \frac{L}{\Gamma^*_{\beta}},
\] (A.12)

and the equilibrium number of suppliers in the economy

\[
S^* \equiv N^*_{\beta} n^* = \left( \frac{L}{\Gamma^*_{\beta}} \right) \left[ \left( \frac{1 - \rho}{\rho} \right) \frac{F}{f} \right]^{1/2}.
\] (A.13)

This completes the characterization of the equilibrium for the case of constant marginal rate of substitution between expected profits and risk.

I now explore the effect of changes in uncertainty on welfare and compare the results to the benchmark model developed in the main text. Using the same definition as before, equilibrium expected welfare is given by

\[
W^* \equiv \frac{\mu n^*_{\beta} \Gamma^*_{\beta}}{\pi^*_{\beta}} = \frac{\mu}{1 + \left( \frac{\sigma^2}{2} \right)^{1/2} \left\{ \left[ (1 - \rho) f \right]^{1/2} + (\rho F)^{1/2} \right\} p^*_{\beta}}.
\] (A.14)

This equation implies that an increase in uncertainty decreases welfare: \( \partial W^*/\partial \sigma^2 < 0 \). This result is the same as in the benchmark model (see Proposition 1). Overall, this appendix shows that the main qualitative properties of the equilibrium are not affected by assuming \( \beta \) is constant.
Across-country correlation

In this section, I extend the model to allow for across-country correlation in suppliers’ shocks. In this case, the correlation between productivity shocks depends on the location of each supplier as follows

\[
\text{corr}(z_s, z_t) = \begin{cases} 
\rho & \text{if } s, t \in S_j, \\
\delta & \text{if } s \in S_j \text{ and } t \in S - S_j,
\end{cases}
\]

(A.15)

where \(\delta \leq \rho\) capture the impact of worldwide shocks on supplier productivity.

As in the benchmark case presented in the main text, the symmetry of suppliers within countries, the absence trade costs, and the fact that countries are identical imply that the optimal number of suppliers is constant across countries and that the optimal number of workers is independent of the supplier's location. Therefore, in this general case, firm profits are also defined as in the closed economy; see equation (5). The key implication of the cross-country correlation is that the variance of profits now depends on the variance of worldwide shocks in addition to the number of countries from which the firm buys inputs

\[
\text{var}(\pi) = \left\{1 - \rho + \frac{n[\rho + (J - 1)\delta]}{J}\right\} n\rho^2 l^2 \sigma^2,
\]

(A.16)

where, as before, \(n\) denotes the total number of suppliers per final good firm. I note that when \(\delta = 0\), the variance of profits reduces to the benchmark case (see equation (30)).

The free-entry equilibrium is characterized by the following first-order and free-entry conditions

\[
\begin{align*}
\frac{\partial \mathbb{E}[U(\pi)]}{\partial n} &= 0 \iff (p - \mu) m - f = (\beta/2) \left\{1 - \rho + \frac{n[\rho + (J - 1)\delta]}{J}\right\} \sigma^2 m^2, \\
\frac{\partial \mathbb{E}[U(\pi)]}{\partial m} &= 0 \iff p - \mu = \beta \left\{1 - \rho + \frac{n[\rho + (J - 1)\delta]}{J}\right\} \sigma^2 m. \\
\mathbb{E}[U(\pi)] &= 0 \iff pmu l - (F + nf + nl) = n\rho^2 l^2 n\left\{1 - \rho + \frac{n[\rho + (J - 1)\delta]}{J}\right\}.
\end{align*}
\]

(A.17)

Combining these conditions, it is straightforward to show that the optimal number of workers per supplier is given by \(l^*\) defined in equation (19) and that the optimal number of suppliers per final good firm is given by

\[
n_{FT}^* = \left[\frac{\rho J}{(J - 1)\delta + \rho}\right]^{\frac{1}{2}} n^*.
\]

(A.18)

As expected, setting \(\delta = 0\) yields the benchmark results presented in the main text (see equation (36)). To summarize, adding worldwide shocks has no impact on the optimal input demand per supplier, but it decreases the optimal number of suppliers per final good firms. In the limit, when \(\delta = \rho\), the number of suppliers becomes equal to the number of suppliers in the closed economy. This happens because, in that case, diversification through international trade does not provide any additional benefit relative to sourcing only domestically.
B Technical Appendix B

B.1 Proof of proposition 1

Part (a): From equations (17) and (19), it follows that
\[
\frac{\partial n^*}{\partial \sigma^2} = 0 \quad \text{and} \quad \frac{\partial l^*}{\partial \sigma^2} = -\frac{1}{2} \left( \frac{l^*}{\sigma^2} \right) < 0.
\] (B.1)
These results show that the number of suppliers per firm is invariant to the variance of productivity shocks, whereas the size of each supplier is decreasing in the variance of productivity shocks.

Part (b): From equations (21) and (22), it follows that
\[
\frac{\partial \Gamma^*}{\partial \sigma^2} = -\frac{1}{2} \left( \frac{\Gamma^*}{\sigma^2} \right < 0 \quad \text{and} \quad \frac{\partial N^*}{\partial \sigma^2} = -L \left( \frac{1}{\Gamma^*} \right) \frac{\partial \Gamma^*}{\partial \sigma^2} > 0.
\] (B.2)
These results show that total employment per firm is decreasing in the variance of productivity shocks, whereas the equilibrium mass of firms is decreasing in the variance of productivity shocks.

Part (c): From the definition of equilibrium price given in equation (20), it follows that
\[
\frac{\partial p^*}{\partial \sigma^2} = \frac{1}{\mu} \left( \frac{1}{2\sigma^2} \right) \left\{ (1 - \rho) f \right\}^{\frac{1}{2}} \left( 1 + \left( \frac{1}{\rho F} \right) \right) > 0.
\] (B.3)
These results show that equilibrium price is increasing in the variance of productivity shocks.

Part (d): From the definition of equilibrium expected welfare given in equation (28), it follows that
\[
\frac{\partial W^*}{\partial \sigma^2} = -\frac{W^2}{\mu} \left( \frac{1}{2\sigma^2} \right) \left\{ (1 - \rho) f \right\}^{\frac{1}{2}} \left( 1 + \left( \frac{1}{\rho F} \right) \right) < 0.
\] (B.4)
These results show that equilibrium expected aggregate welfare is decreasing in the variance of productivity shocks.

B.2 Proof of proposition 2

From the trade share equation (41), it follows that
\[
\frac{\partial \theta_{FT}^*}{\partial \sigma^2} = -\frac{J}{J - 1} \left( \frac{1}{p_{FT}^*} \right)^2 \frac{\partial p_{FT}^*}{\partial \sigma^2} < 0,
\] (B.5)
where the inequality comes from the fact that
\[
\frac{\partial p^*}{\partial \sigma^2} = \frac{1}{\mu} \left( \frac{1}{2\sigma^2} \right) \left\{ (1 - \rho) f \right\}^{\frac{1}{2}} \left( 1 + \left( \frac{1}{\rho F} \right) \right) > 0.
\] (B.6)
This last result is a generalization of equation (B.3).

### B.3 Proof of proposition 3

**Part (a):** From equation (35), the number of suppliers per final good firm is given by
\[ n_{FT}^* = J^{\frac{1}{2}} n^* , \]
where \( n^* \) denotes the optimal number of suppliers under autarky defined in equation (17). It follows that \( n_{FT}^* > n^* \) \( \forall J \geq 2 \), such that a move from autarky \( (J = 1) \) to free trade \( (J \geq 2) \) increases the total number of suppliers from which a firm sources.

In the free trade equilibrium, final good firms will distribute their suppliers evenly across symmetric countries. From equation (35), this implies that each final good firm will contract with \( n_{FT}^* / J = J^{-\frac{1}{2}} n^* \) suppliers from each country, where \( n^* \) denotes the optimal number of (domestic) suppliers under autarky. Because \( J^{-\frac{1}{2}} < 1 \) \( \forall J \geq 2 \), moving from autarky to free trade decreases the number of suppliers located in the domestic country.

**Part (b):** From equations (21) and (38), it follows that
\[ \Gamma_{FT}^* - \Gamma^* = \left( J^{\frac{1}{2}} - 1 \right) \left\{ \left( \frac{1 - \rho}{\rho} \right) fF \right\}^{\frac{1}{2}} + \left( \frac{2F}{\rho \sigma^2} \right)^{\frac{1}{2}} > 0, \]
where the inequality comes from the fact that \( J^{1/2} > 1 \) \( \forall J \geq 2 \). This establishes that \( \Gamma_{FT}^* > \Gamma^* \). This result implies that
\[ N_{FT}^* = \frac{L}{\Gamma_{FT}^*} < N^* = \frac{L}{\Gamma^*} , \]
which proves that the mass of final good firms is smaller in the free trade equilibrium than in the closed economy equilibrium.

**Part (c):** From equations (20) and (37), it follows that
\[ p_{FT}^* - p^* = \frac{2\sigma^2}{2\mu} \left( J^{-\frac{1}{2}} - 1 \right) (\rho F)^{\frac{1}{2}} < 0, \]
where the inequality comes from the fact that \( J^{-1/2} < 1 \) \( \forall J \geq 2 \). Therefore moving from autarky to free trade decreases the equilibrium price of final goods.

From equations (5) and (37), it follows that
\[ \pi_{FT}^* - \pi^* = \left( \frac{1 - \rho}{\rho} fF \right)^{\frac{1}{2}} \left( J^{\frac{1}{2}} - 1 \right) > 0, \]
where the inequality comes from the fact that \( J^{\frac{1}{2}} > 1 \) \( \forall J \geq 2 \). Therefore moving from autarky to free trade increases equilibrium profits per firm.

**Part (d):** By definition, output per worker is equal to welfare. Proposition 3 shows that welfare is greater under free trade than under autarky. Therefore, output per worker is greater under free trade than under autarky.
B.4 Proof of proposition 4

From the definition of closed economy and free trade equilibrium welfare defined in equations (28) and (40), respectively, it follows that

\[ \frac{\mu}{W^*_{FT}} - \frac{\mu}{W^*} = \left( \frac{\sigma^2}{2} \right)^{\frac{1}{2}} \left( J^{\frac{1}{2}} - 1 \right) (\rho F)^{\frac{1}{2}} < 0. \]

This result implies that \( W^*_{FT} > W^* \), such that a move from autarky to free trade increases welfare. Further, from equation (40) it follows that

\[ \frac{\partial W^*_{FT}}{\partial J} > 0, \]

which demonstrates that the gains from trade are increasing in the number of countries in the world.

B.5 Proof of proposition 5

Part (a): The optimal number of domestic and foreign suppliers are implicitly defined by equations (48) and (49). Combining these equations, totally differentiating with respect to \( n_D \) and \( \tau \), and rearranging yields

\[ \frac{\partial n_D}{\partial \tau} = \frac{1}{\rho} \left[ \frac{(1-\rho)w}{2f\sigma^2} \right]^{\frac{1}{2}} \left\{ 1 + \left\{ \frac{1}{J-1} \left[ \frac{(1-\rho)}{\rho} \right] \frac{F}{f} - n_D^2 \right\}^{-\frac{1}{2}} \frac{n_D}{J-1} \right\}^{-1} > 0. \]

The term in curly brackets is equal to the number of domestic suppliers as defined in (49), so it is positive in equilibrium. This equation shows that the optimal number of domestic suppliers is increasing in trade costs, \( \tau \).

Similarly, inverting (48) and (49) to obtain \( n_D = \left[ \gamma^{-1}(n_X) \right] \) and \( n_D = \gamma^{-1}(n_X) \), combining the equations, totally differentiating with respect to \( n_X \) and \( \tau \), and rearranging yields

\[ \frac{\partial n_X}{\partial \tau} = -\frac{1}{\rho} \left[ \frac{(1-\rho)w}{2f\sigma^2} \right]^{\frac{1}{2}} \left\{ 1 + \left[ \frac{(1-\rho)}{\rho} \right] \frac{F}{f} - (J-1)n_X^2 \right\}^{-\frac{1}{2}} (J-1)n_X^{-1} < 0. \]

The last term in square brackets is equal to the number of foreign suppliers as defined in (49), so it is positive in equilibrium. This equation shows that the optimal number of foreign suppliers is decreasing in trade costs.

Part (b): From equation (47), the optimal demand per foreign supplier is equal to the ratio of optimal demand per domestic supplier over \textit{ad valorem} trade barriers. From equation (15), the optimal quantity of inputs demanded from each domestic supplier is independent of trade costs. It follows that

\[ \frac{\partial l_X}{\partial \tau} = -\frac{l^*}{\tau^2} < 0. \]

This equation shows that the optimal quantity of inputs demanded from each foreign supplier is decreasing in trade costs.
B.6 Proof of proposition 6

**Part (a):** The optimal number of domestic and foreign suppliers are implicitly defined by equations (48) and (49). Combining these equations, totally differentiating with respect to \( n_D \) and \( \sigma^2 \), and rearranging yields

\[
\frac{\partial n_D}{\partial \sigma^2} = \frac{\tau - 1}{2 \rho \sigma^2} \left[ \frac{(1 - \rho)}{2 f \sigma^2} \right]^{\frac{1}{2}} \left\{ 1 + \left[ \frac{1}{J - 1} \left[ \frac{(1 - \rho)}{\rho} \frac{F}{J} - n_D^2 \right] \right]^{-\frac{1}{2}} \frac{n_D}{J - 1} \right\}^{-1} < 0.
\]

This equation shows that the optimal number of domestic suppliers is increasing in the variability of materials production costs \( \sigma^2 \), a measure of uncertainty.

Similarly, inverting (48) and (49) to obtain \( n_D = (-1)(n_X) \) and \( n_D = \gamma^{-1}(n_X) \), combining the equations, totally differentiating with respect to \( n_X \) and \( \tau \), and rearranging yields

\[
\frac{\partial n_X}{\partial \sigma^2} = \frac{\tau - 1}{2 \rho \sigma^2} \left[ \frac{(1 - \rho)}{2 f \sigma^2} \right]^{\frac{1}{2}} \left\{ 1 + \left[ \frac{1}{J - 1} \left( \frac{1 - \rho}{\rho} \frac{F}{J} - (J - 1)n_X^2 \right) \right]^{-\frac{1}{2}} (J - 1)n_X \right\}^{-1} > 0.
\]

This equation shows that the optimal number of foreign suppliers is increasing in the variability of materials production costs \( \sigma^2 \).

**Part (b):** From equation (15), the derivative of the optimal quantity of inputs demanded from each domestic supplier with respect to the variance of production costs is given by

\[
\frac{\partial l_D}{\partial \sigma^2} = -\frac{l^*}{2 \sigma^2} < 0.
\]

This result shows that the optimal quantity of materials demanded from each domestic supplier is decreasing in the variability of materials production costs \( \sigma^2 \), a measure of uncertainty.

From equation (47), the optimal demand per foreign supplier is equal to the ratio of optimal demand per domestic supplier over ad valorem trade barriers. Therefore, the derivative of the optimal quantity of inputs demanded from each foreign supplier with respect to the variance of production costs is given by

\[
\frac{\partial l_X}{\partial \sigma^2} = \frac{1}{\tau} \frac{\partial l_D}{\partial \sigma^2} = -\frac{l^*}{2 \sigma^2} < 0.
\]

This result shows that the optimal quantity of materials demanded from each foreign supplier is decreasing in uncertainty.
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<tr>
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<th>List of countries in samples</th>
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Notes: The table lists the countries included in the full sample. A * indicates that the country appears in the World Bank’s Exporter Dynamics Dataset.